NH: PT's big brother

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but if $\boldsymbol{H}$ has PT symmetry, e.g. $V(\boldsymbol{r}) = V^*(-\boldsymbol{r})$, then some, or in special cases all, energies can be real

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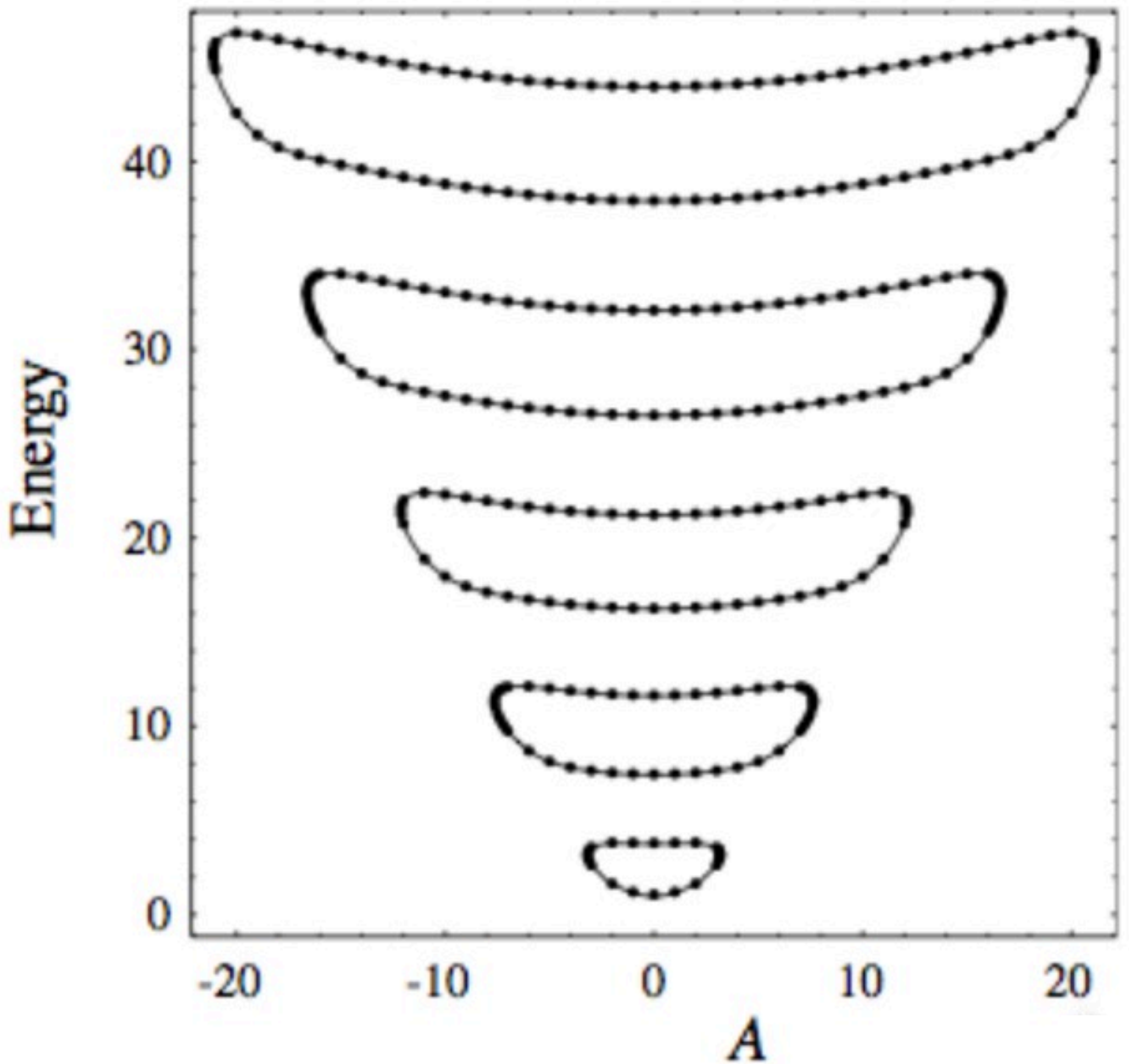

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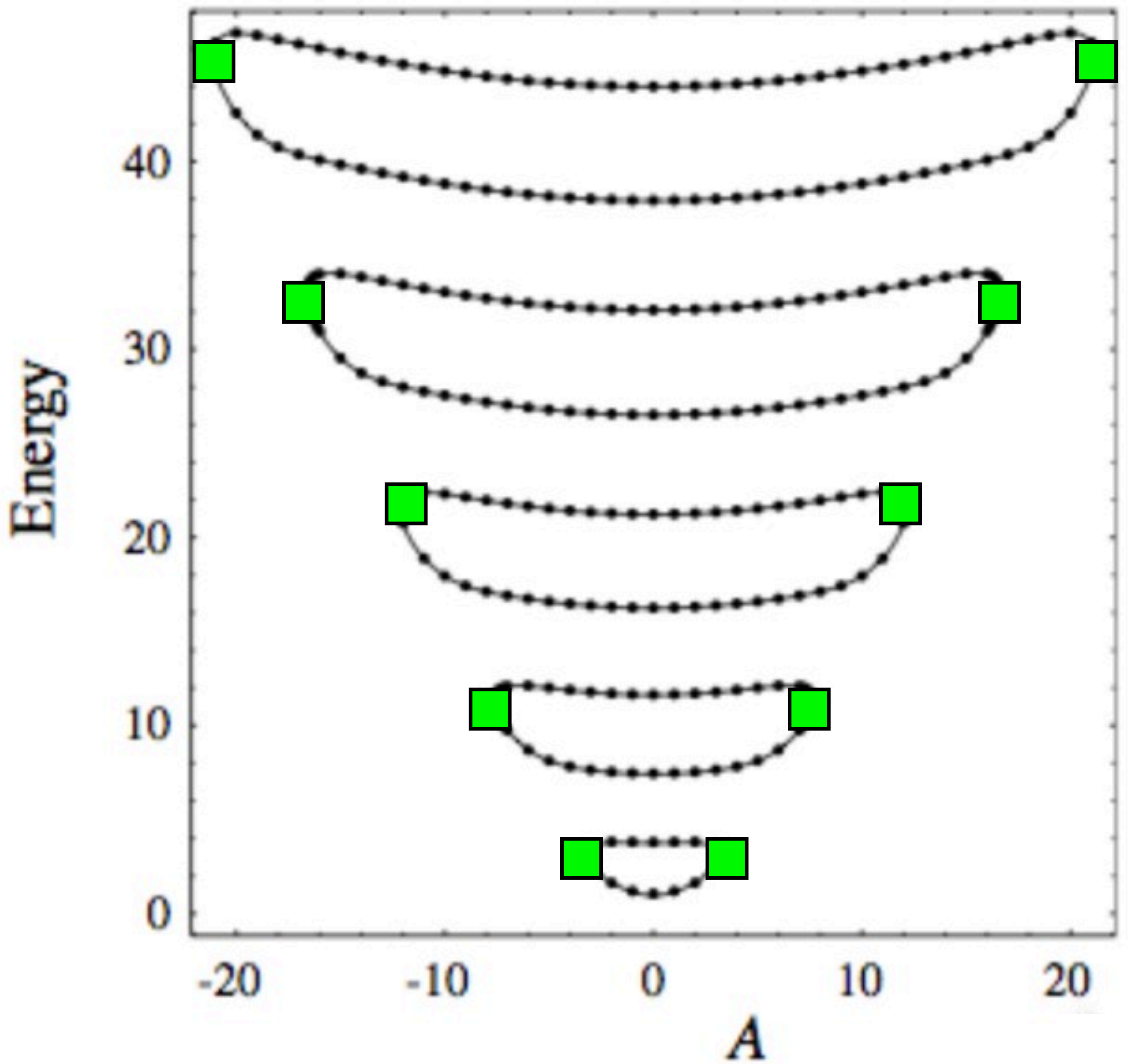

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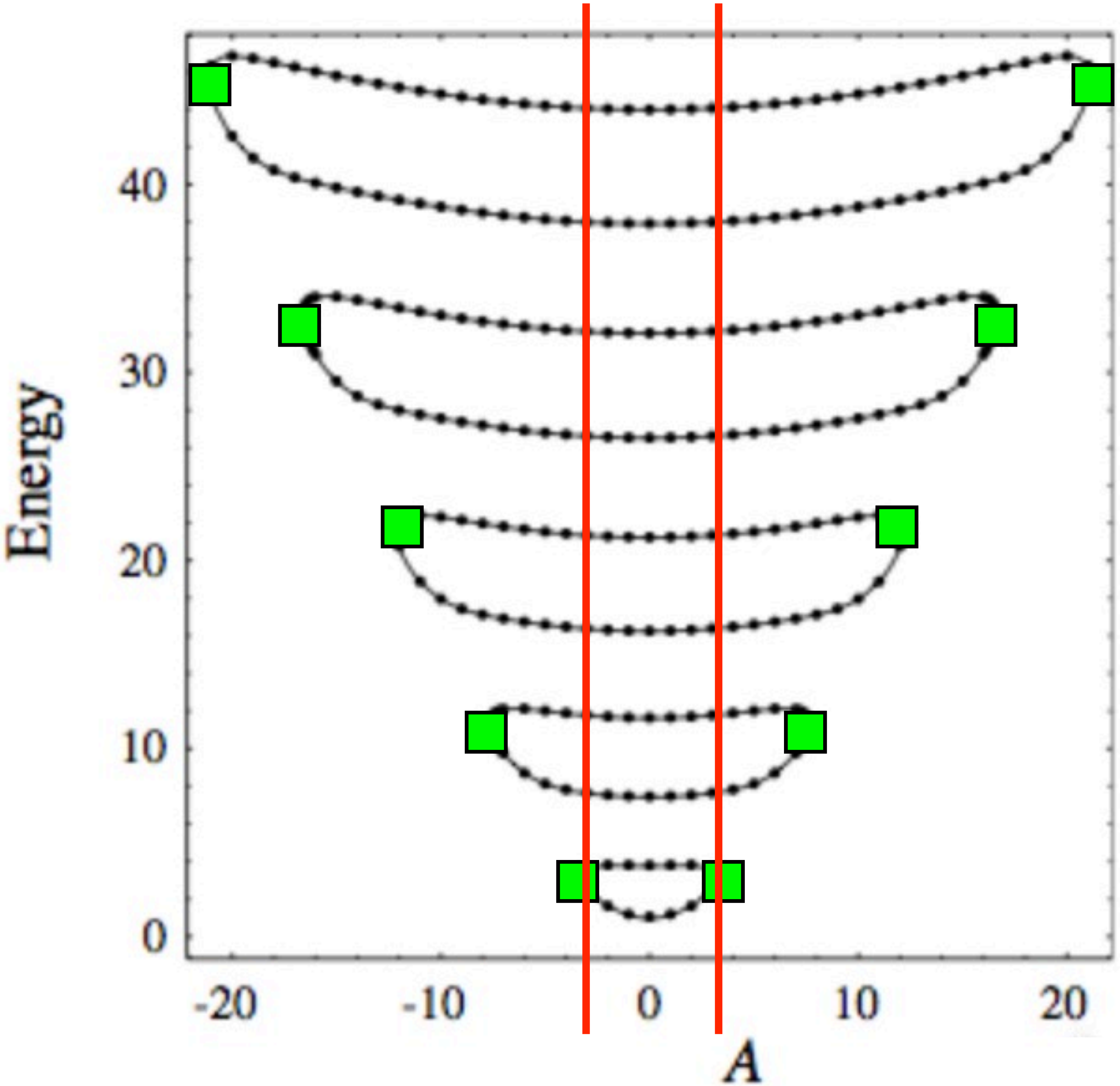

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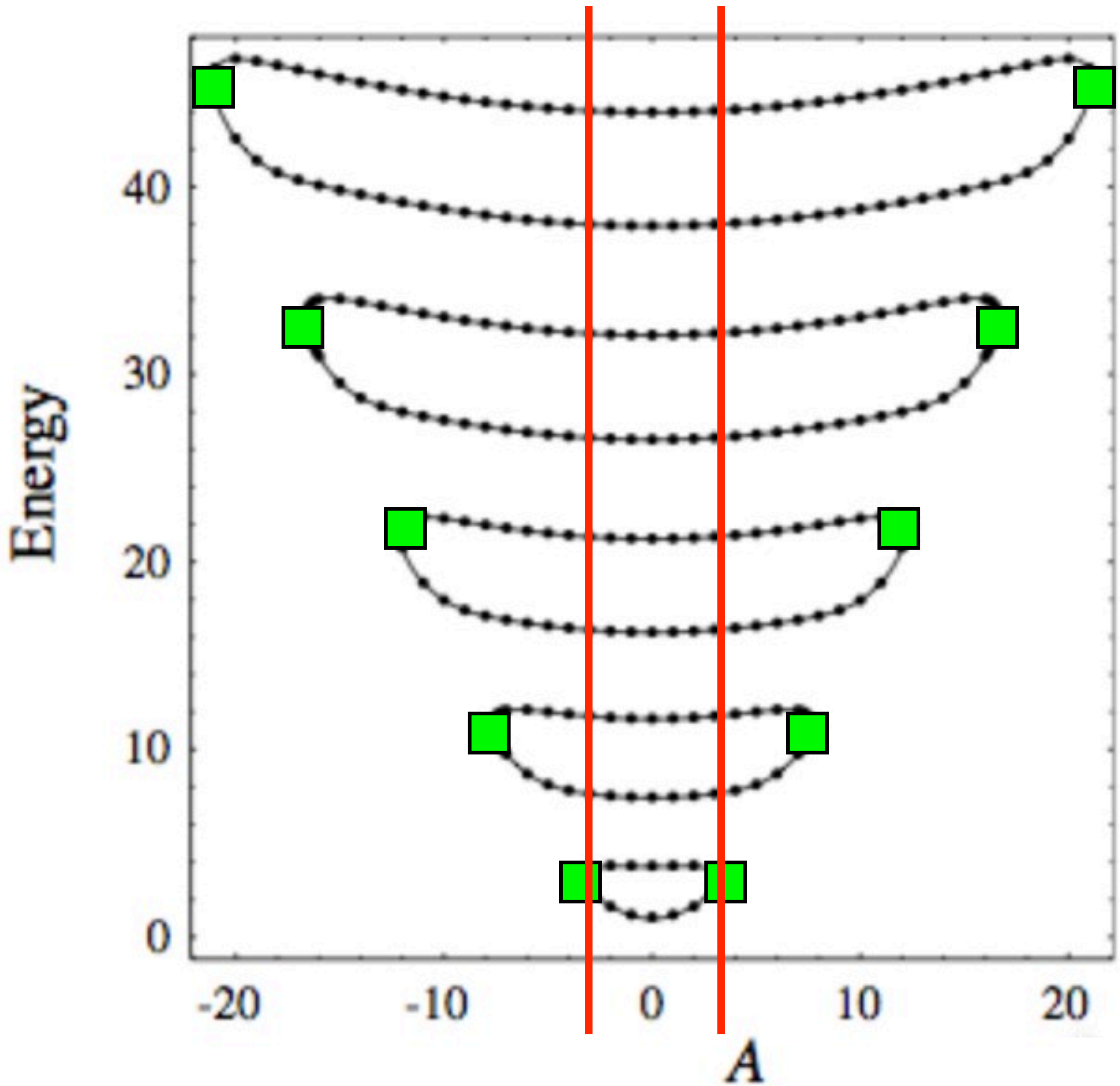

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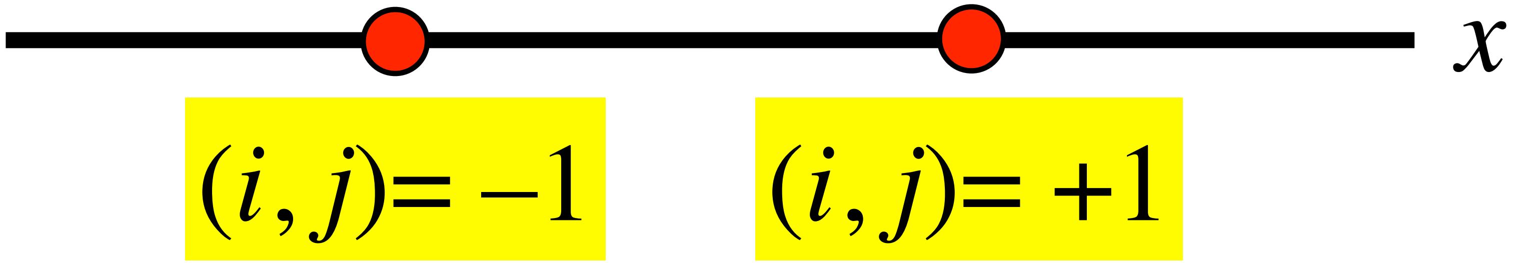
why are ***any*** energies real?

2x2 case: two points on x axis



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$$E = \mathrm{Re}\,a \pm \sqrt{|b|^2 - (\mathrm{Im}\,a)^2}$$ NHDs where $|b| = |\mathrm{Im}\,a|$

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start from orthonormal basis of states $|n\rangle$

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$$|n_A\rangle \equiv |n\rangle + \boldsymbol{A}|n\rangle$$, satisfying $$\boldsymbol{A}|n_A\rangle = |n_A\rangle$$

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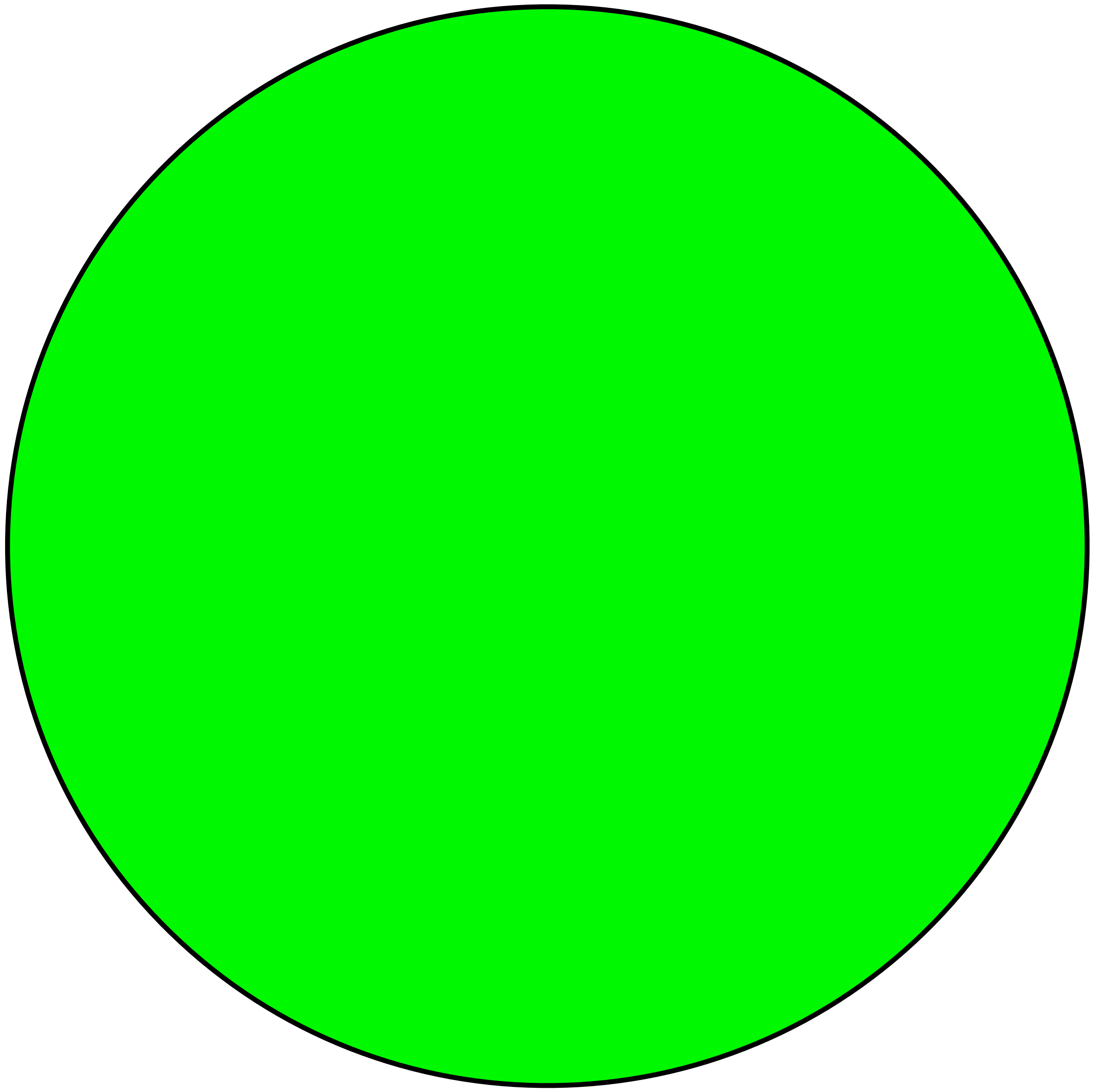

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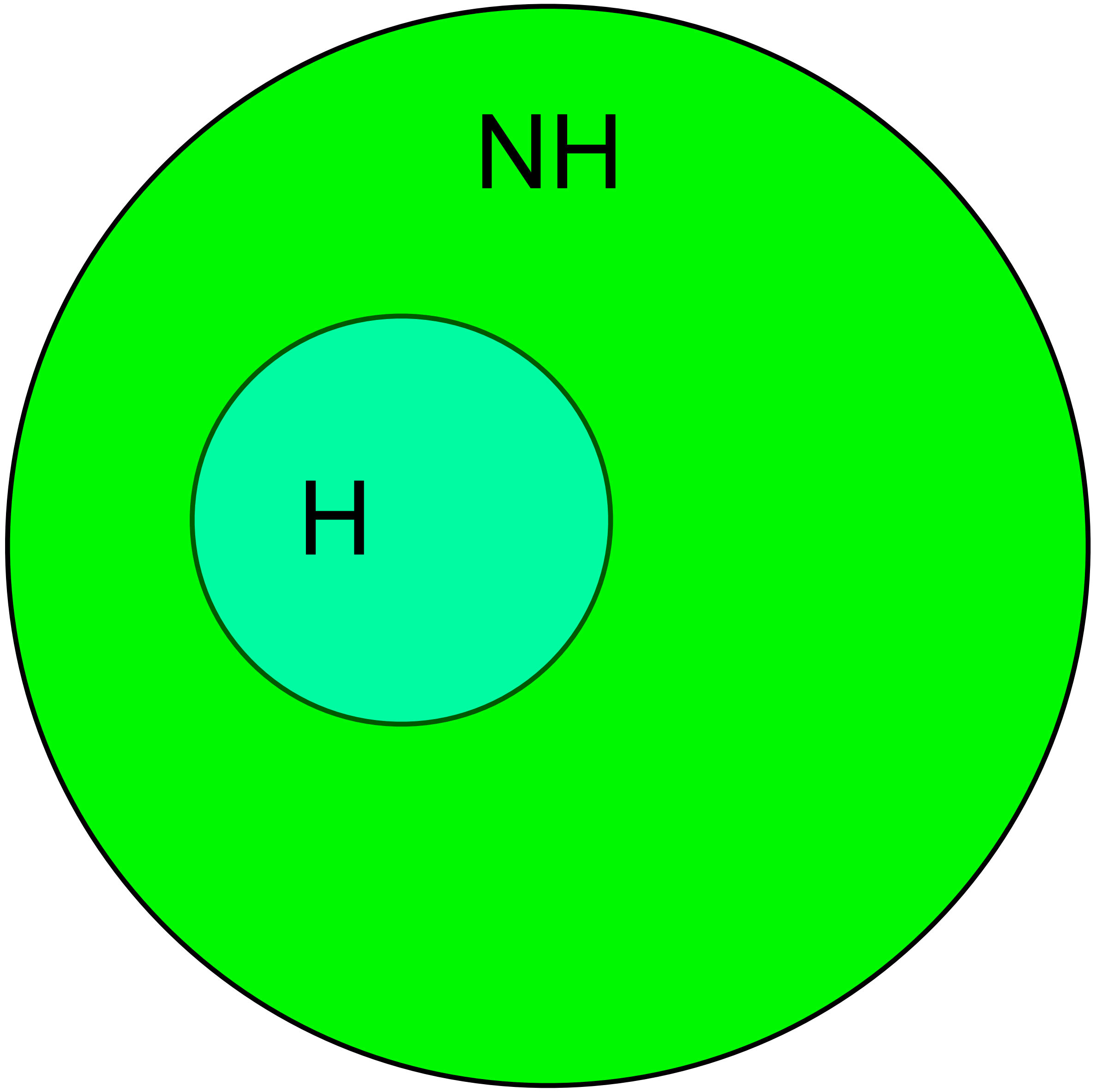

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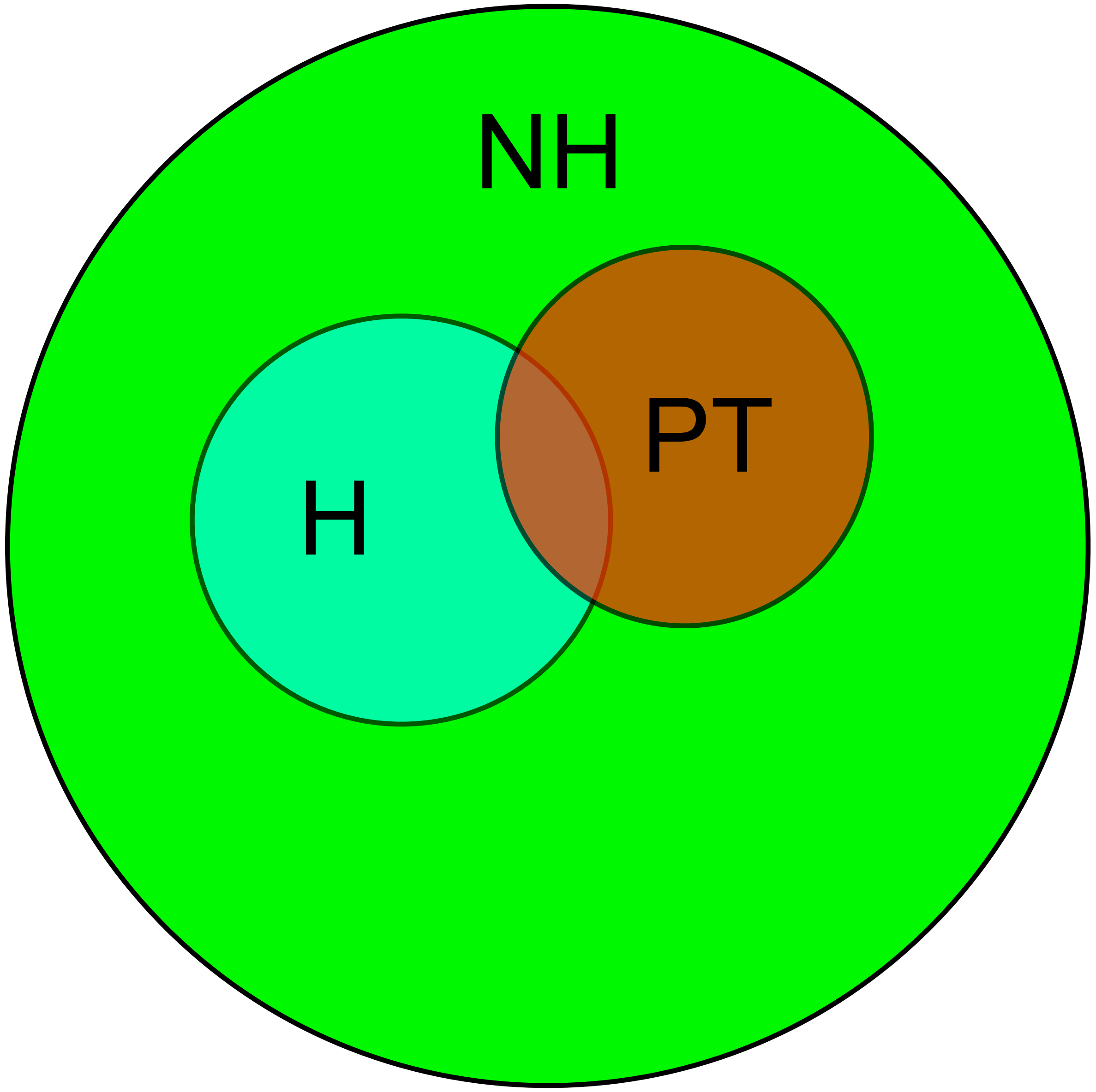

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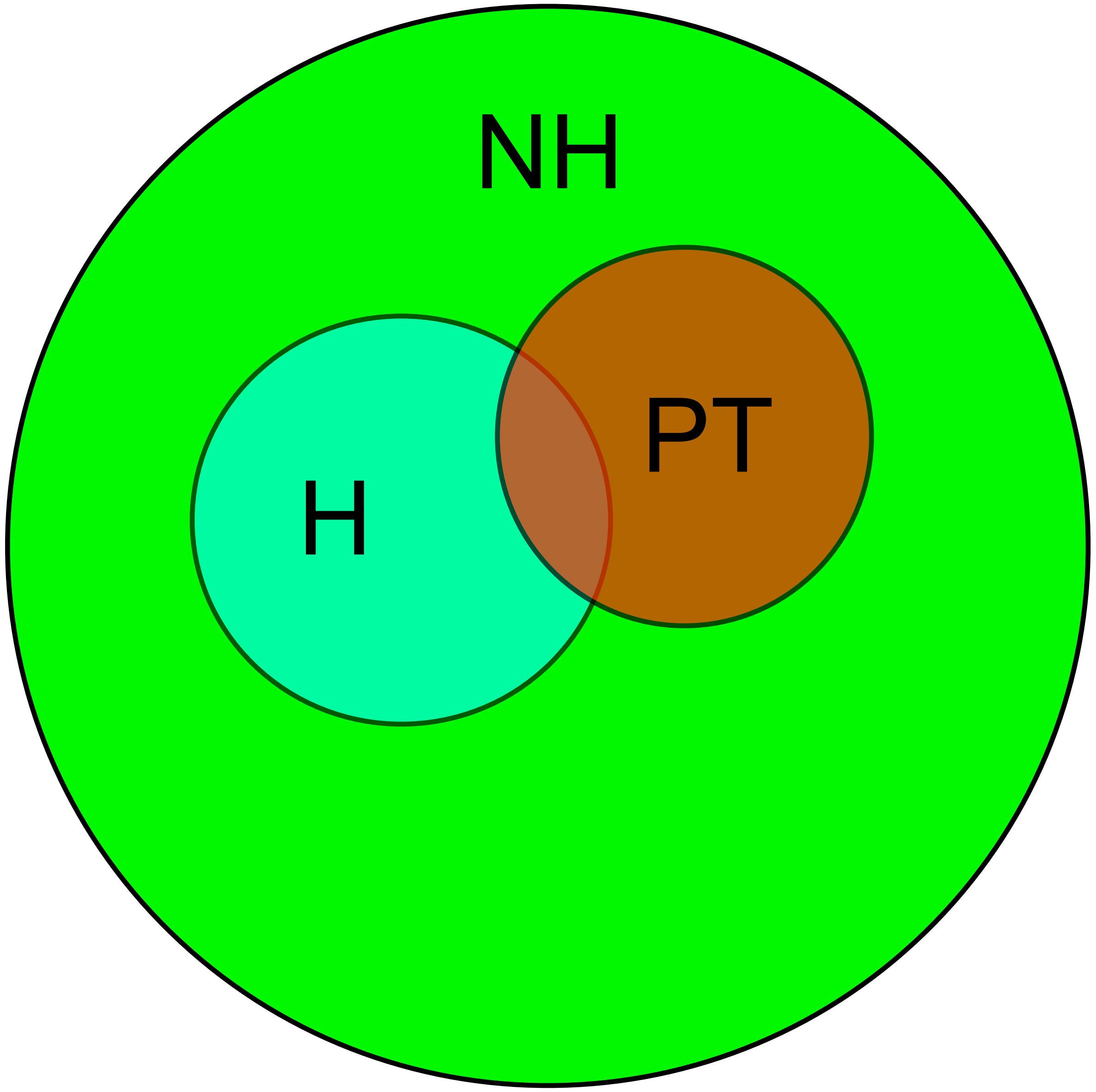

importance of NH,
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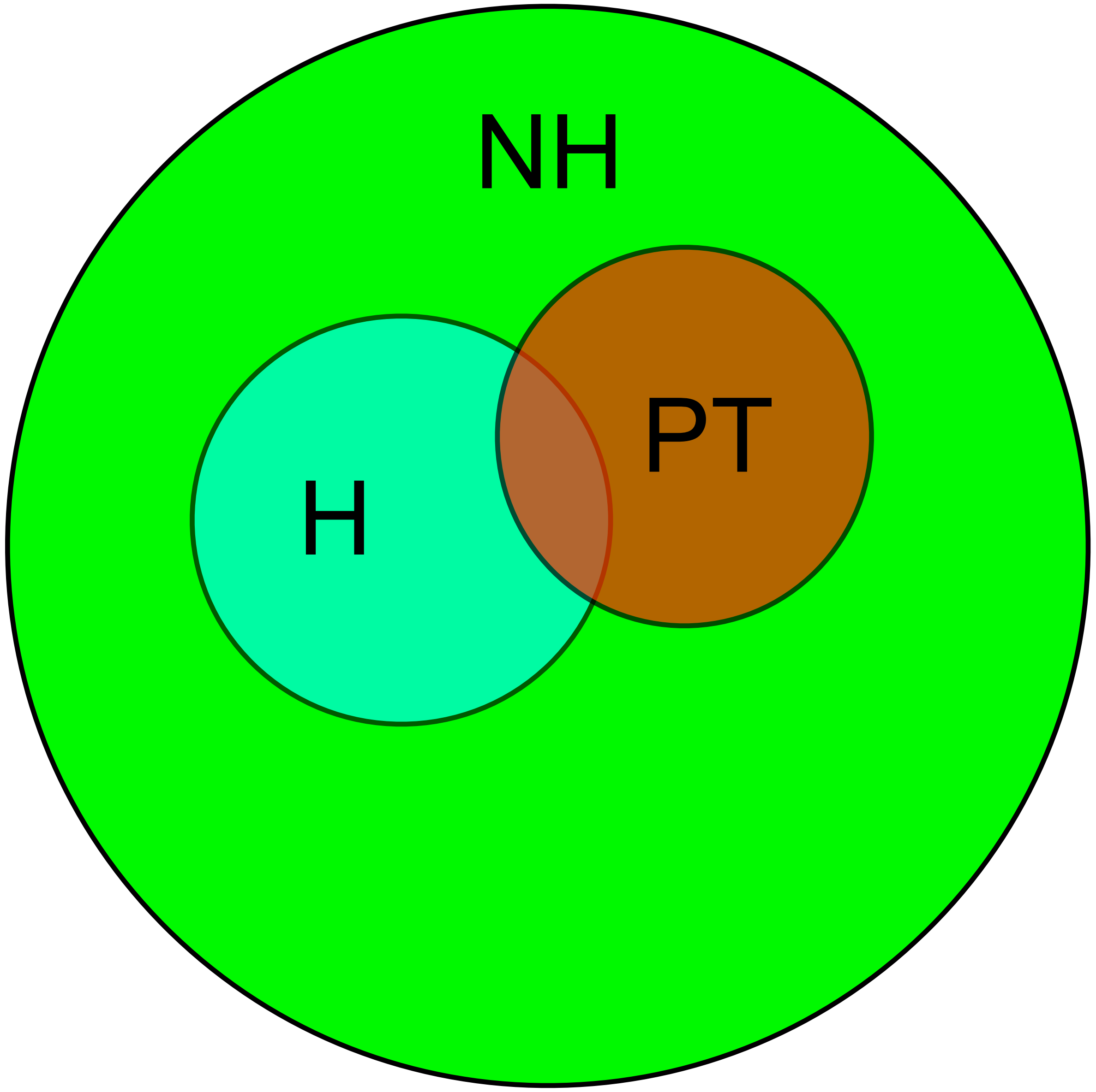

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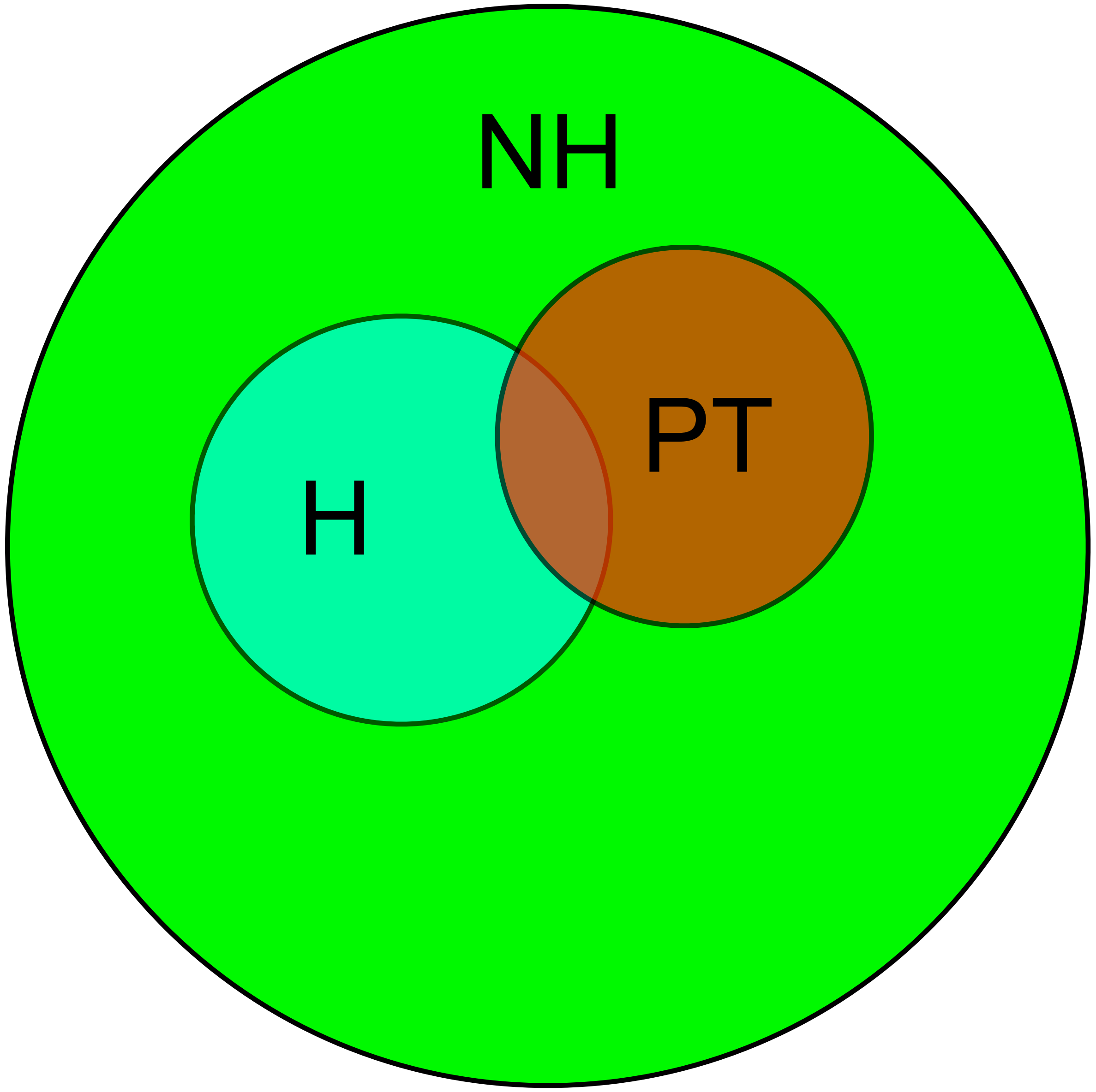

importance of NH, contrasting views

1. NH not fundamental, merely describing decay (or, more recently, gain) associated with freedoms we cannot measure or choose to ignore

2. NH more fundamental than H, which perpetrates the fiction of the isolated system, ignoring the fact that any probing of a system involves coupling it with something else

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counter-counter view 2: examples showing that the new PT scalar product does not represent physics - probability not conserved

diffraction of waves: light, electrons, atoms...

by waves: sound, crystals, light...

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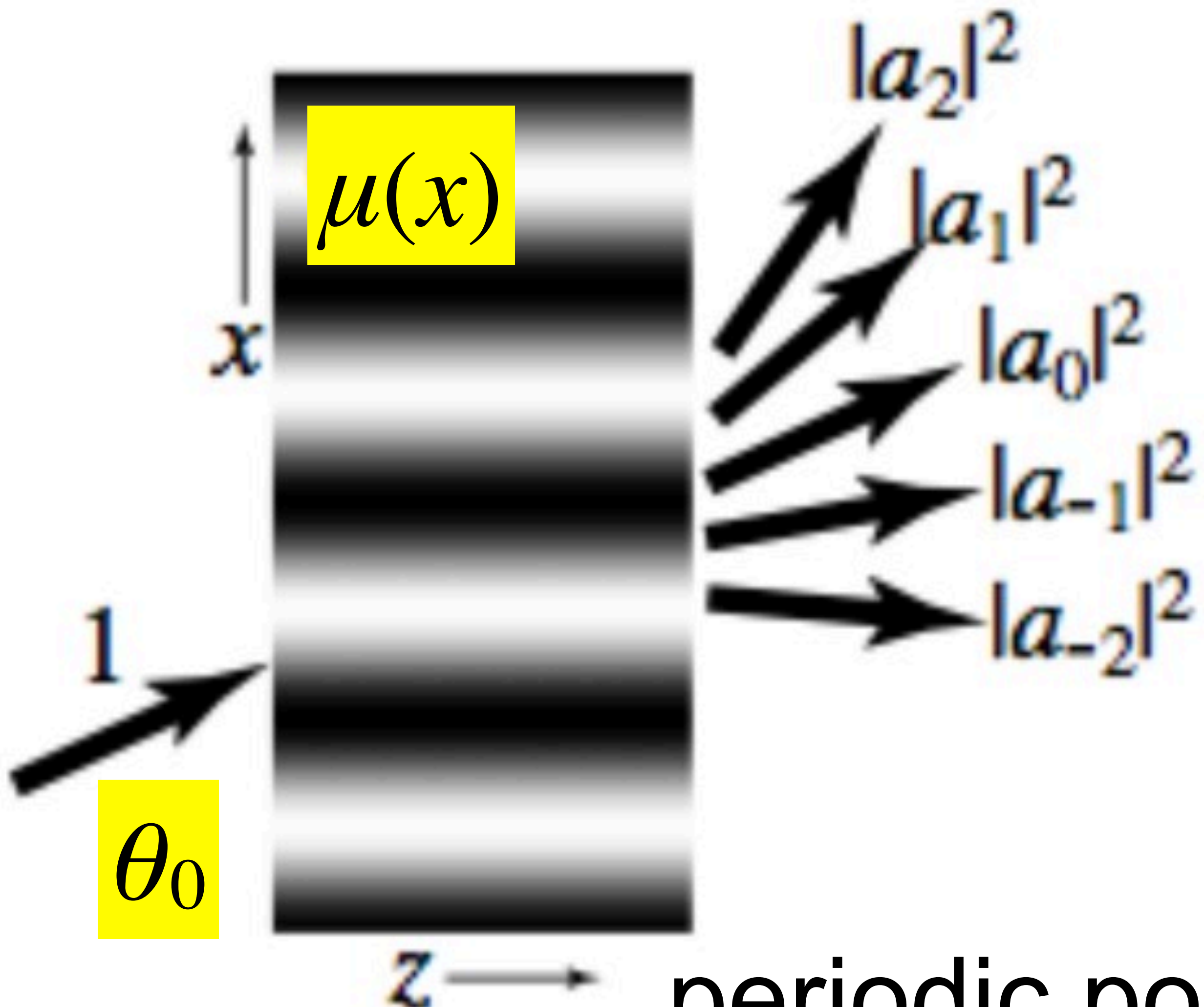

Bragg-diffracted beam intensities $|a_n|^2$

periodic potential (refractive index)^2 $\mu(x)$

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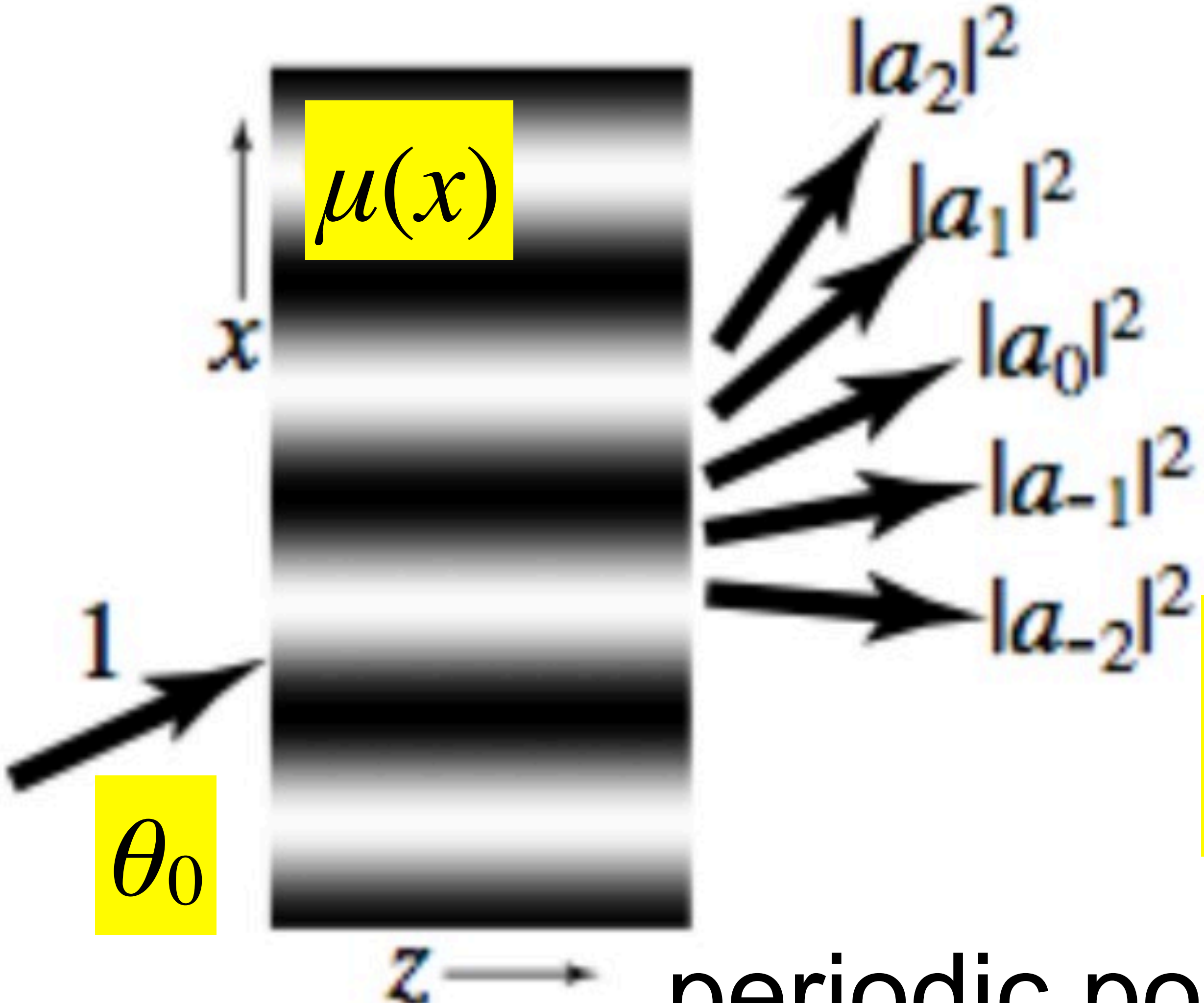

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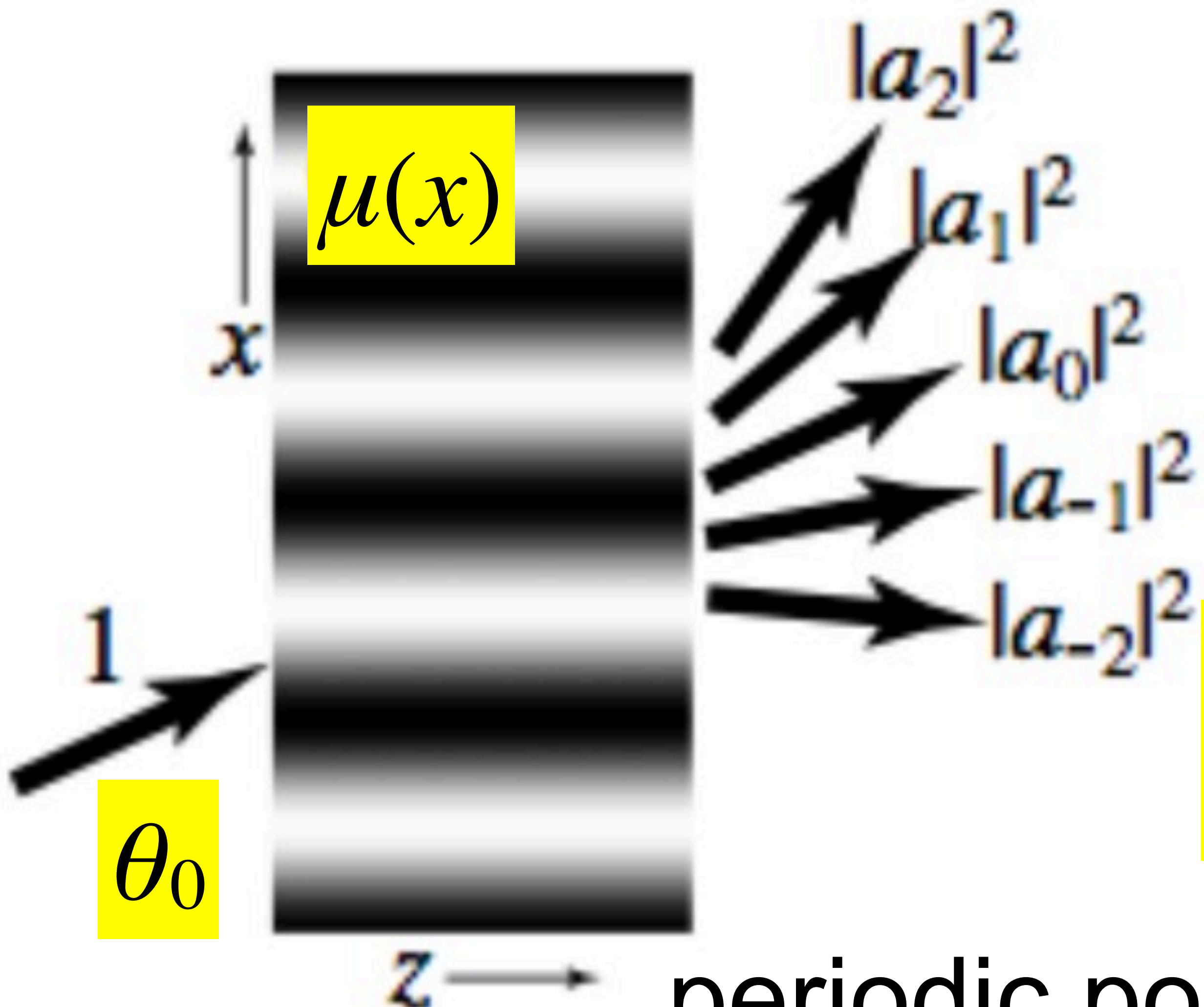

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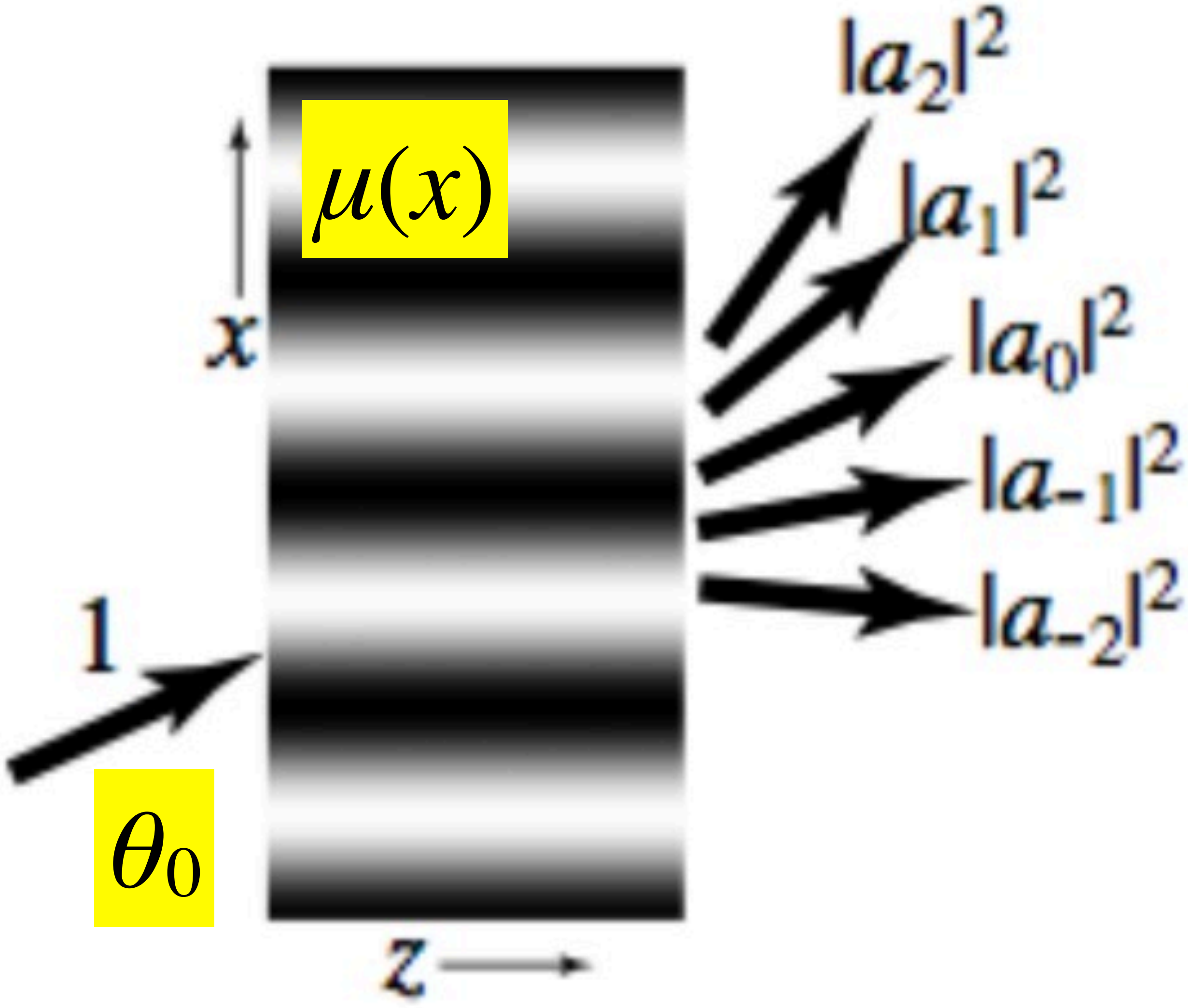
PT symmetric if

$$\mu(x) = \mu_\mathrm{h}(x) + \mu_\mathrm{a}(x)$$

$$\mu_\mathrm{h}(x)\ \text{(hermitian) real even}$$

$$\mu_\mathrm{a}(x)\ \text{(antihermitian) imaginary odd}$$

total emergent intensity (current, probability, Poynting)



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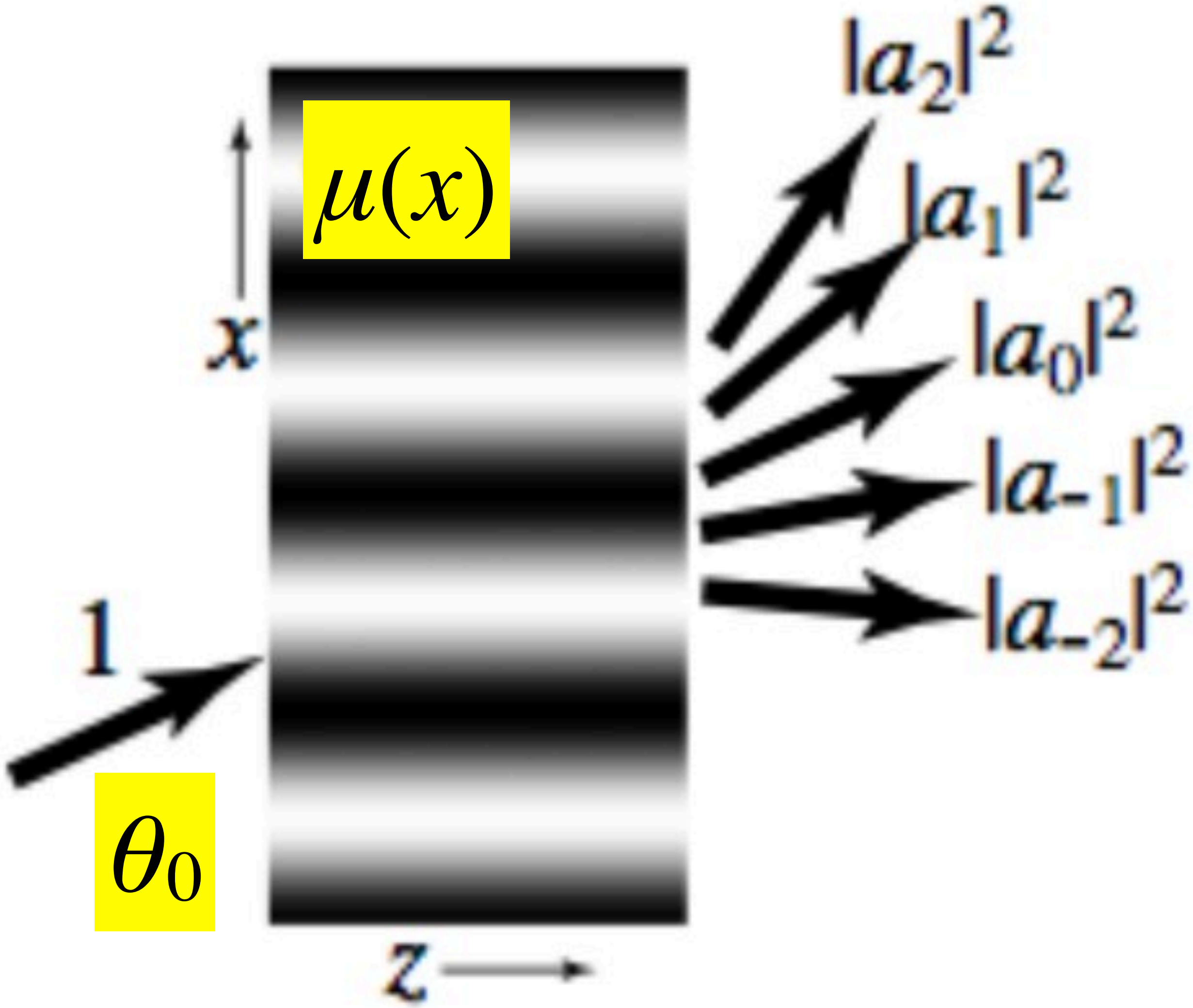

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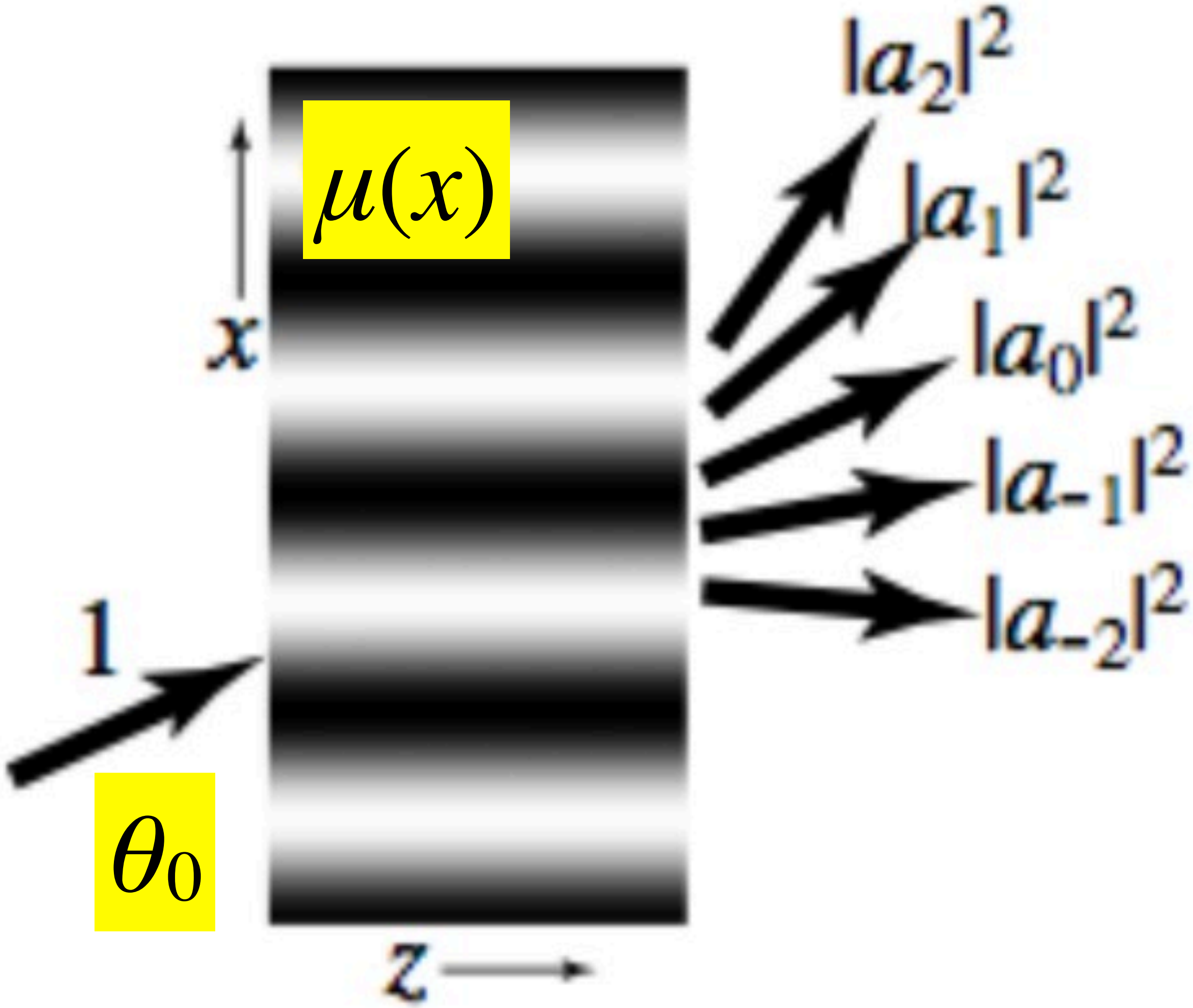

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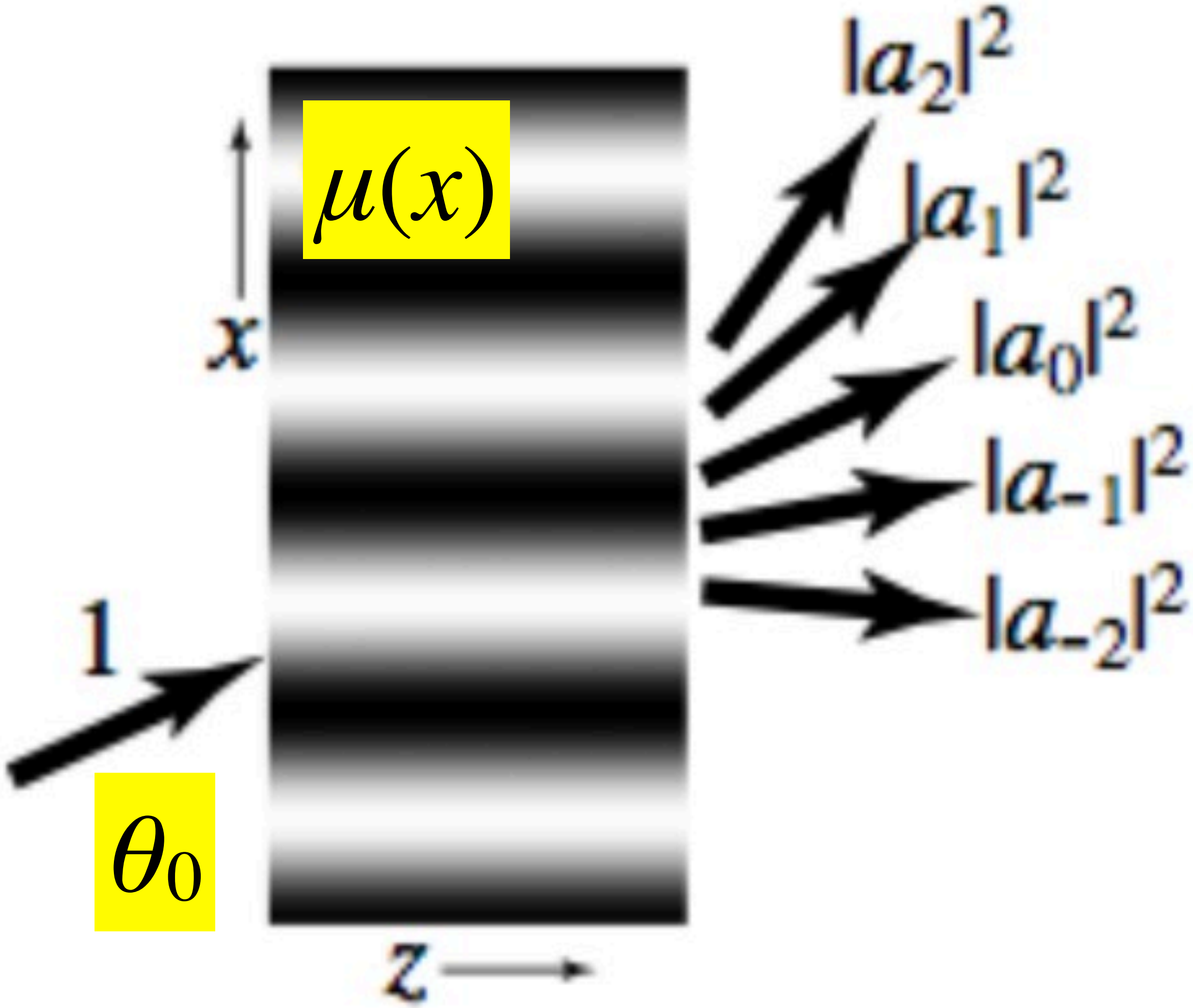

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No!

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Optical lattices with PT symmetry are not transparent

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for PT $\mu_n = $ real, $\mu_{\mathrm{h}n} = \mu_{\mathrm{h},-n}$, $\mu_{\mathrm{a}n} = -\mu_{\mathrm{a},-n}$

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beam amplitude evolution

$$\mathrm{i}\partial_z a_n(z) = (n+\alpha_0)^2 a_n(z) + \sum_{m=-\infty}^{\infty} \mu_{n-m} a_m(z), \quad a_n(0) = \delta_{n,0}$$

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$$\longrightarrow \quad \partial_z I(z) = 2\,\mathrm{Im} \sum_{n=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} \mu_{n-m,a} a_n^* a_m$$

paraxial wave equation $$\mathrm{i}\partial_z\psi=-\partial_x^2\psi+\mu(x)\psi$$

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$$\mathrm{i}\partial_z a_n(z)=(n+\alpha_0)^2a_n(z)+\sum_{m=-\infty}^{\infty}\mu_{n-m}a_m(z),\quad a_n(0)=\delta_{n,0}$$

→ $$\partial_z I(z)=2\,\mathrm{Im}\sum_{n=-\infty}^{\infty}\sum_{m=-\infty}^{\infty}\mu_{n-m,a}a_n^*a_m$$ =0 in hermitian case, otherwise not

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PT crystals not transparent, i.e. PT $\not\to$ unitarity

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more general intensity sum rules

$$S \equiv \sum_{n=-\infty}^{\infty} S_n \left| a_n(z) \right|^2 = 1, \quad S_n \text{ real}$$

PT crystals not transparent, i.e. PT $\not\rightarrow$ unitarity

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more general intensity sum rules

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$$I(z) = \sum_{n=-\infty}^{\infty} \left|a_n(z)\right|^2 = 1 + 2\sum_{n=-\infty}^{\infty} \left|a_{2n+1}(z)\right|^2 \geq 1$$

not transparent, gain dominates loss

example 2, interpolating between sum rules

$$\mu(x) = \mu_1 \exp(\mathrm{i}x) + \mu_{-1} \exp(\text{-i}x) = 2\mu_\mathrm{h} \cos x + 2\mathrm{i}\,\mu_\mathrm{a} \sin x$$

(pure trigonometric)

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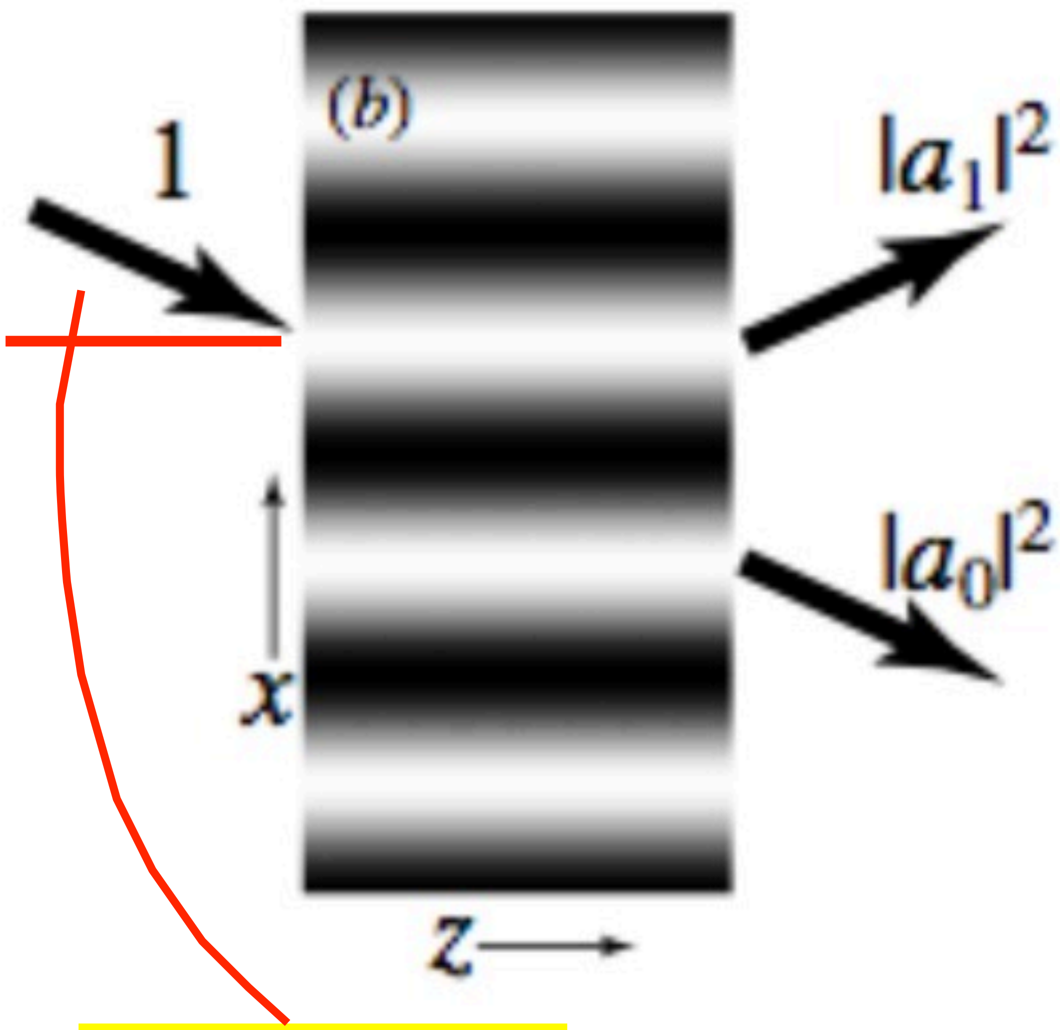
limits

$$\mu_{a1} \to 0, \quad I(z) = \sum_{n=-\infty}^{\infty} |a_n(z)|^2 = 1$$

$$\mu_{h1} \to 0, \quad S(z) = \sum_{n=-\infty}^{\infty} (-1)^n |a_n(z)|^2 = 1$$

example 3: two-beam case, $\left|\mu_h\right| << 1, \quad \left|\mu_a\right| << 1$



$\theta_0 = -\frac{1}{2} + \delta$: deviation from Bragg angle

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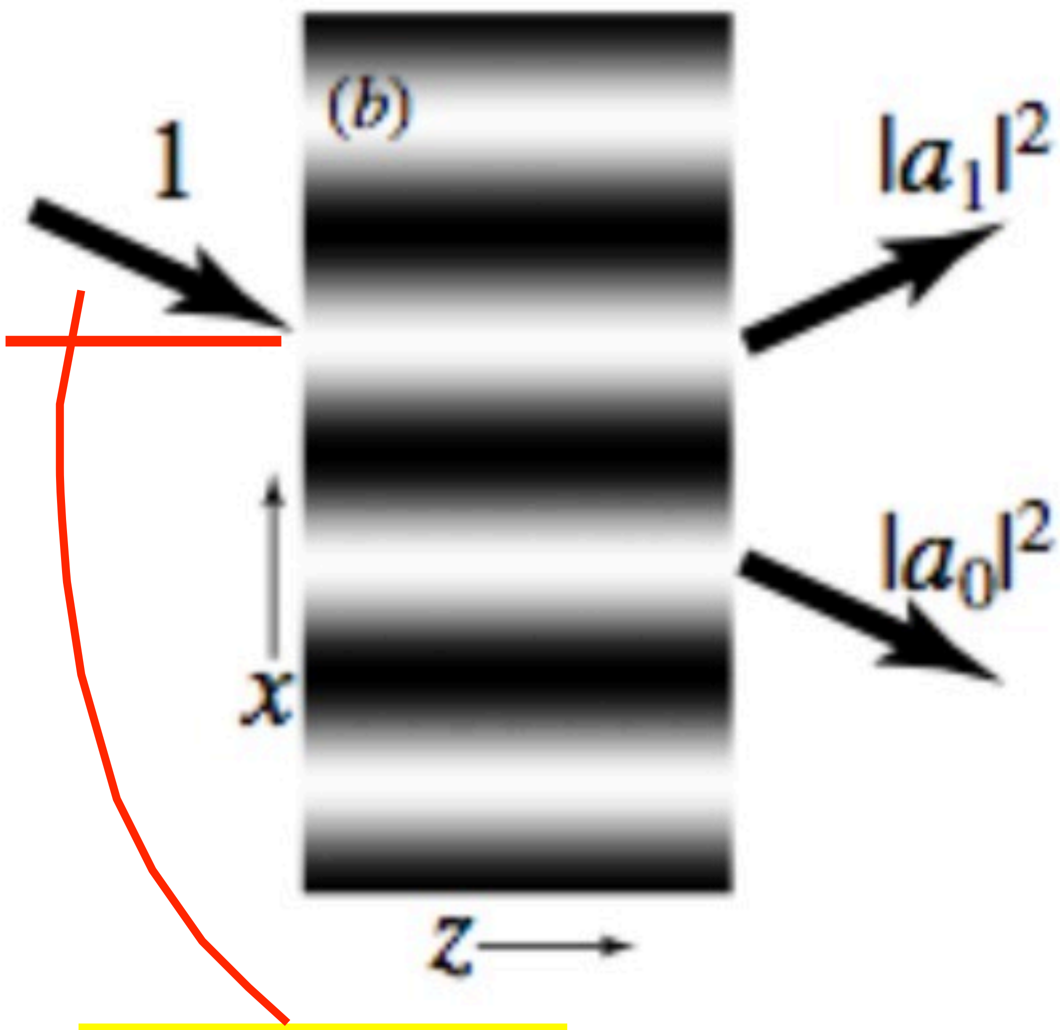

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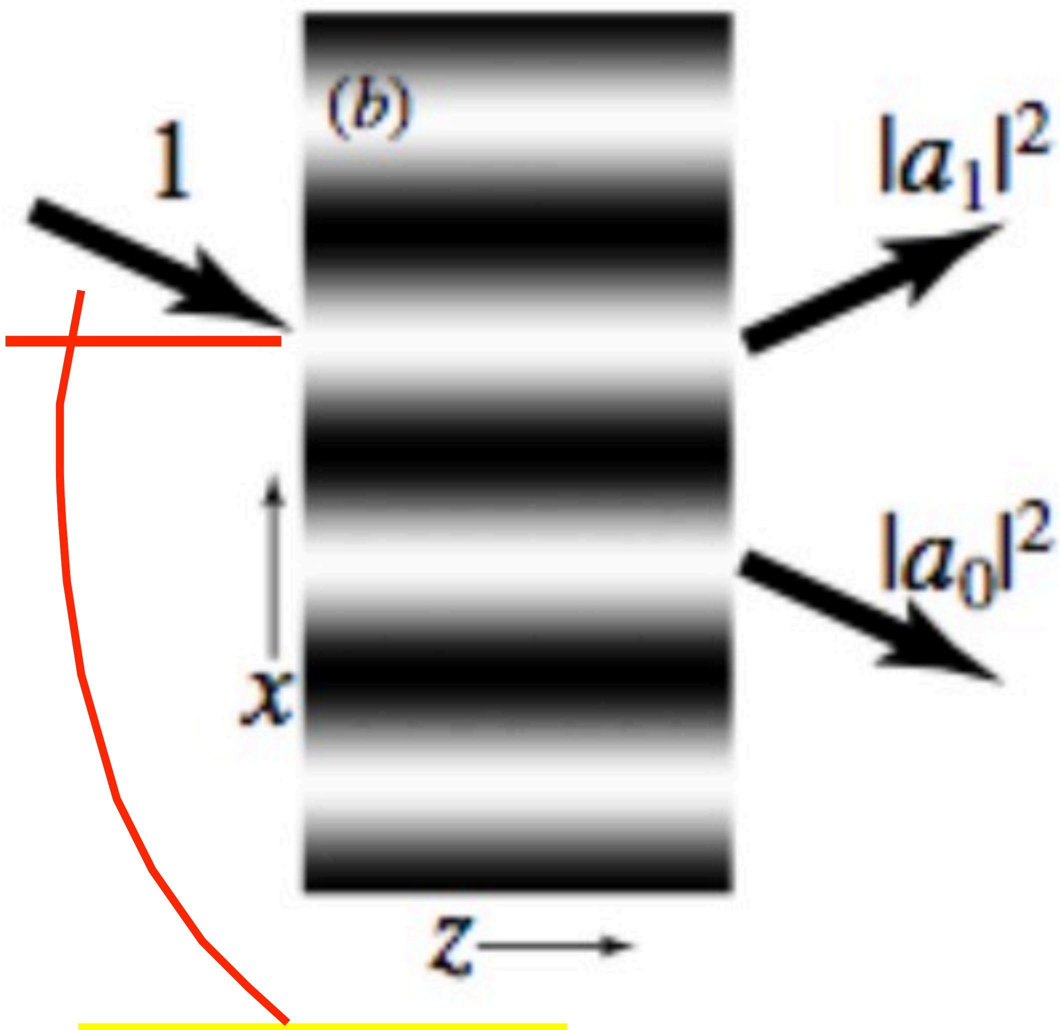

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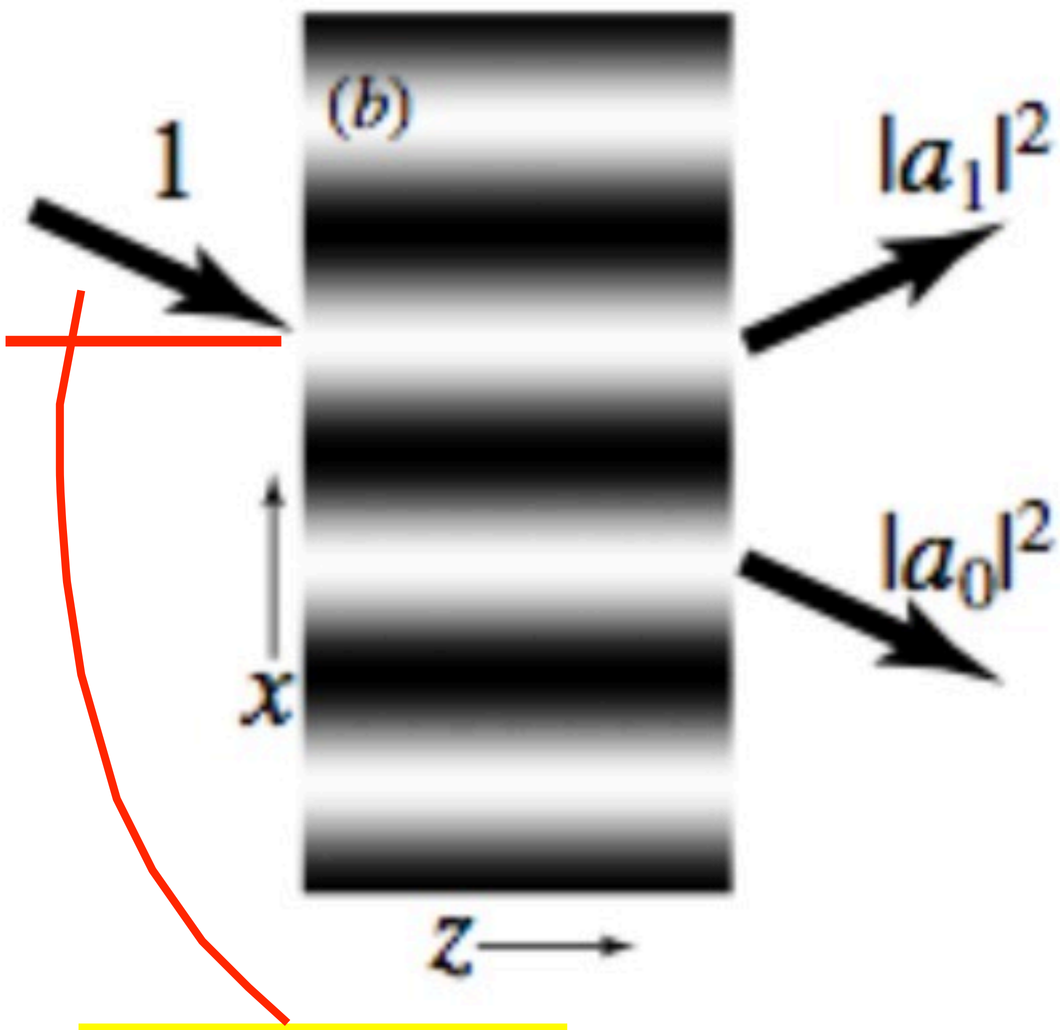

intensity depends on balance of μ_h and μ_a

$$I(z) = 1 + \frac{2\left|a_1(z)\right|^2 \mu_a}{\mu_h + \mu_a}$$

loss if $\frac{\mu_a}{\mu_h} < 0$ and $\left|\mu_a\right| < \left|\mu_h\right|$

gain otherwise

$\theta_0 = -\frac{1}{2} + \delta$: deviation from Bragg angle

$$I_0(z) = |a_0|^2 = \cos^2\left(z\sqrt{\delta^2 + \mu_h^2 - \mu_a^2}\right) + \delta^2 \frac{\sin^2\left(z\sqrt{\delta^2 + \mu_h^2 - \mu_a^2}\right)}{\delta^2 + \mu_h^2 - \mu_a^2}$$

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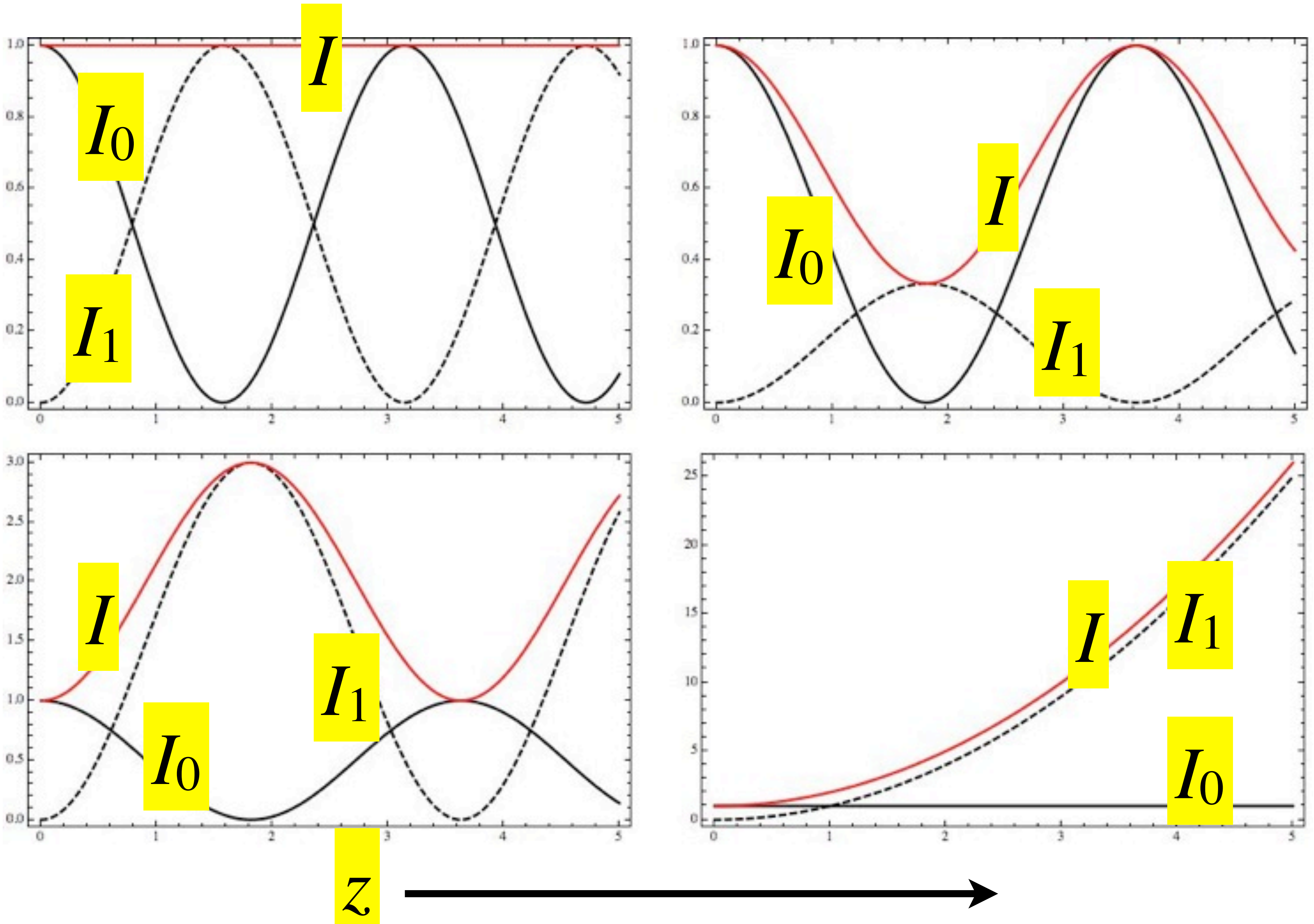

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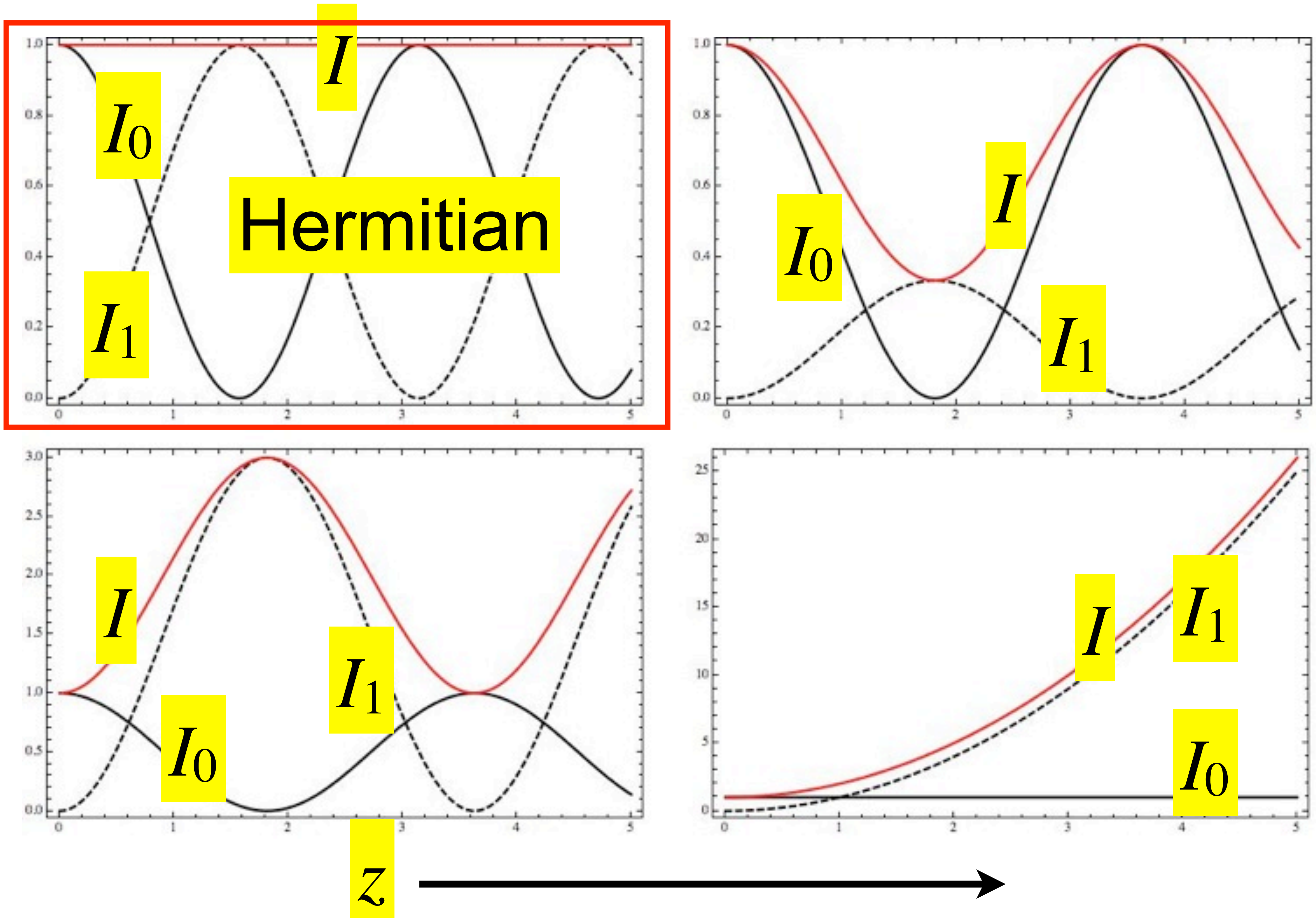

H: $\delta = 0$,

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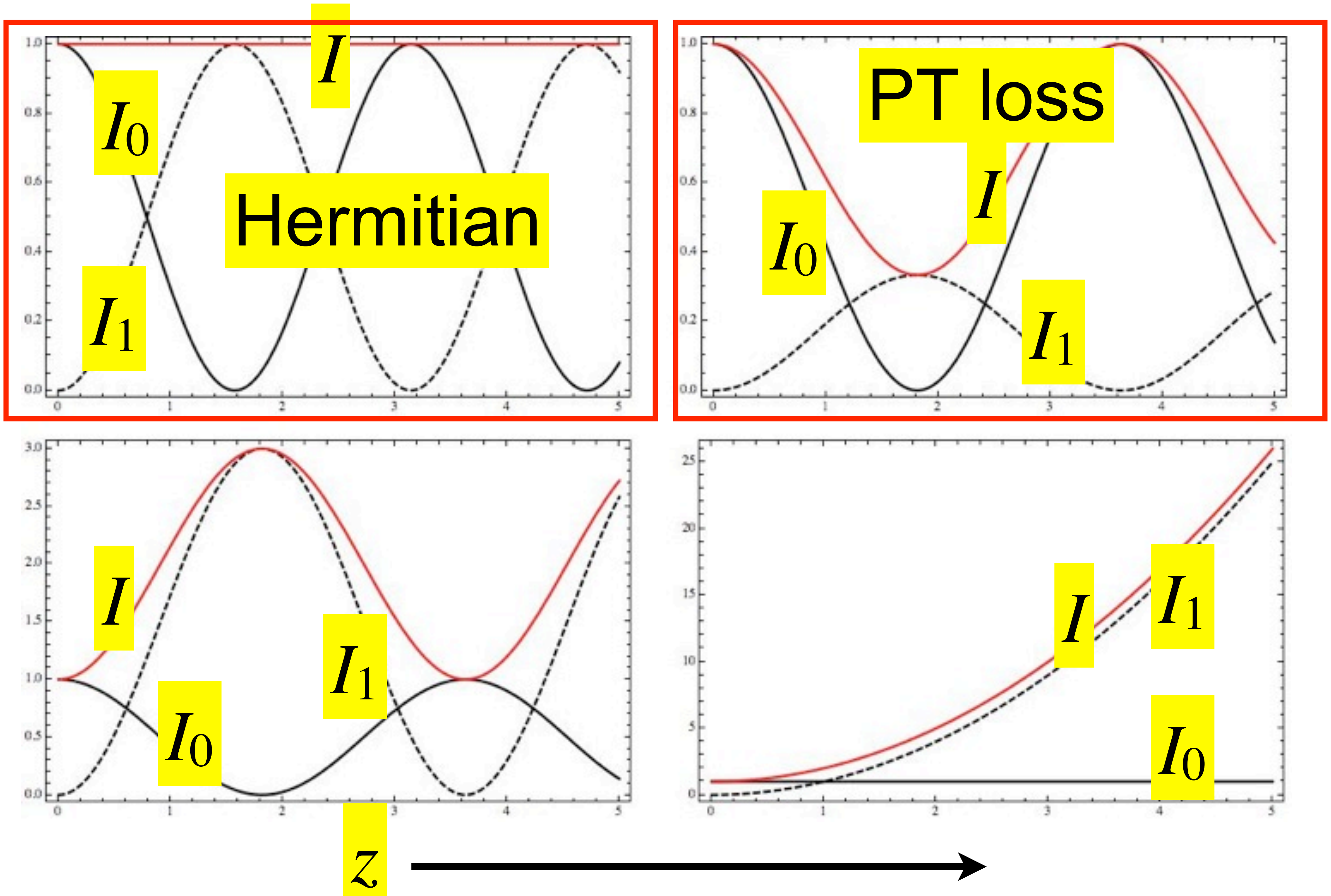

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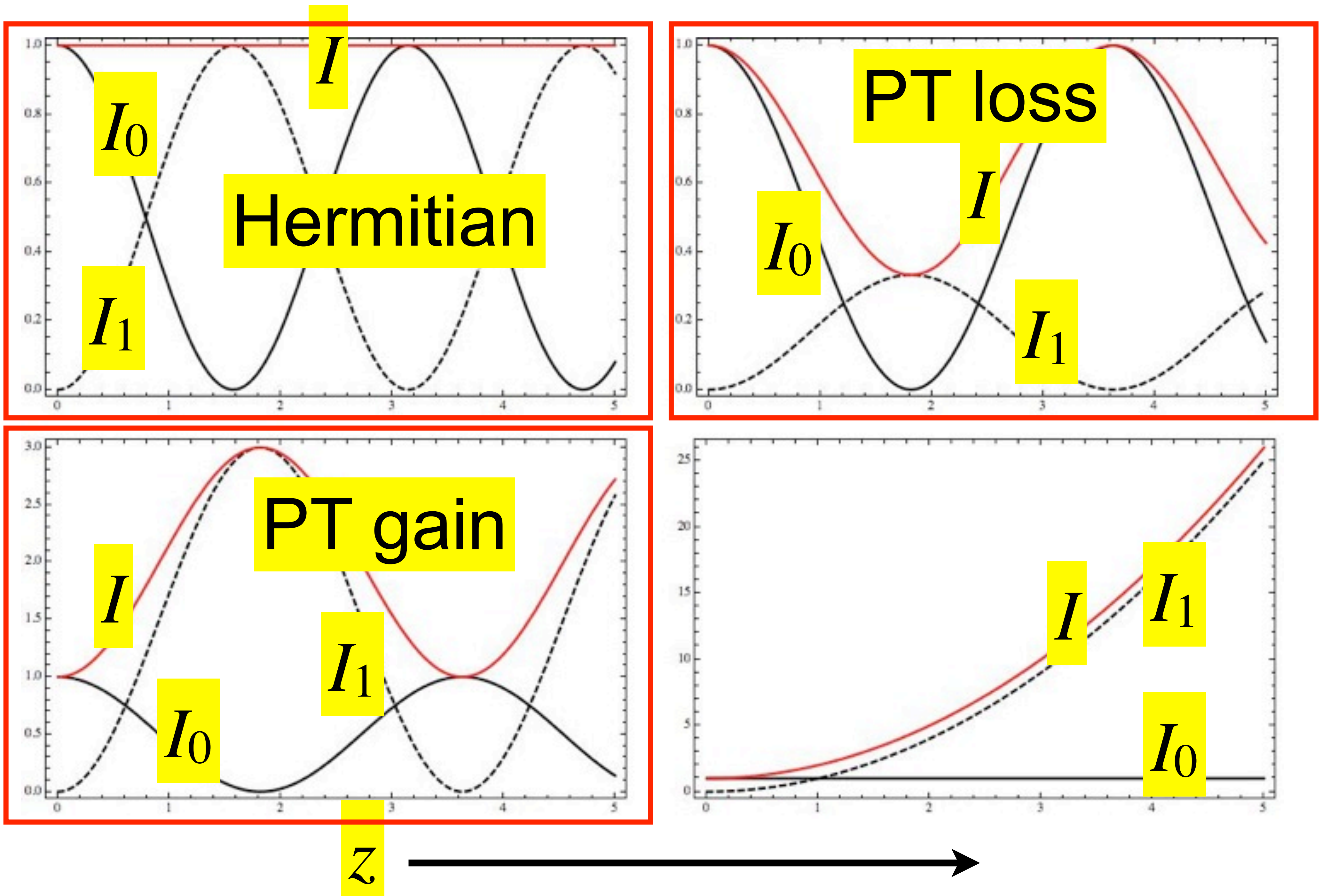

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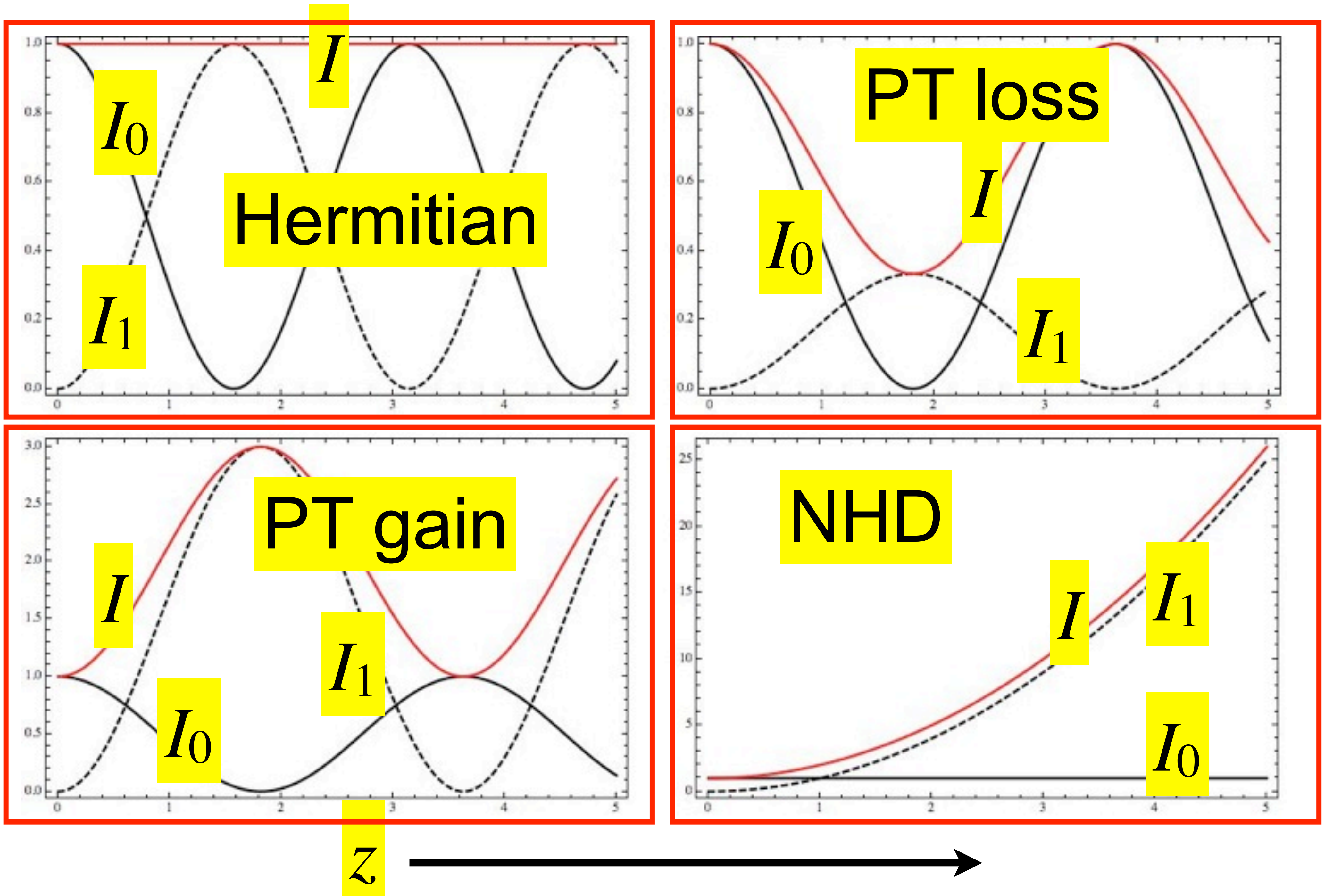

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NHD : $\delta = 0$, $\mu_h = 0.5, \mu_a = 0.5$

NHD case:

$$\delta = \sqrt{\mu_a^2 - \mu_h^2}$$

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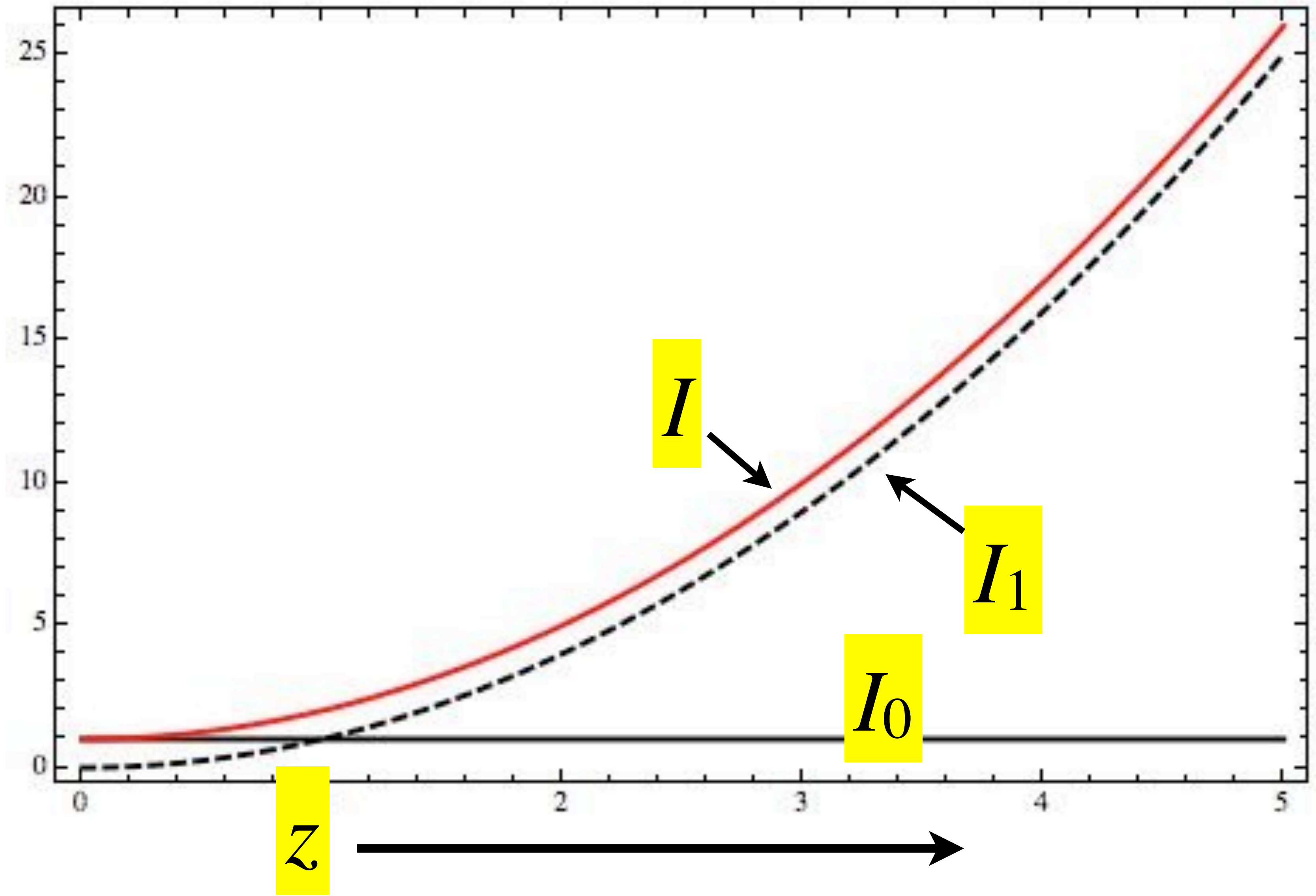

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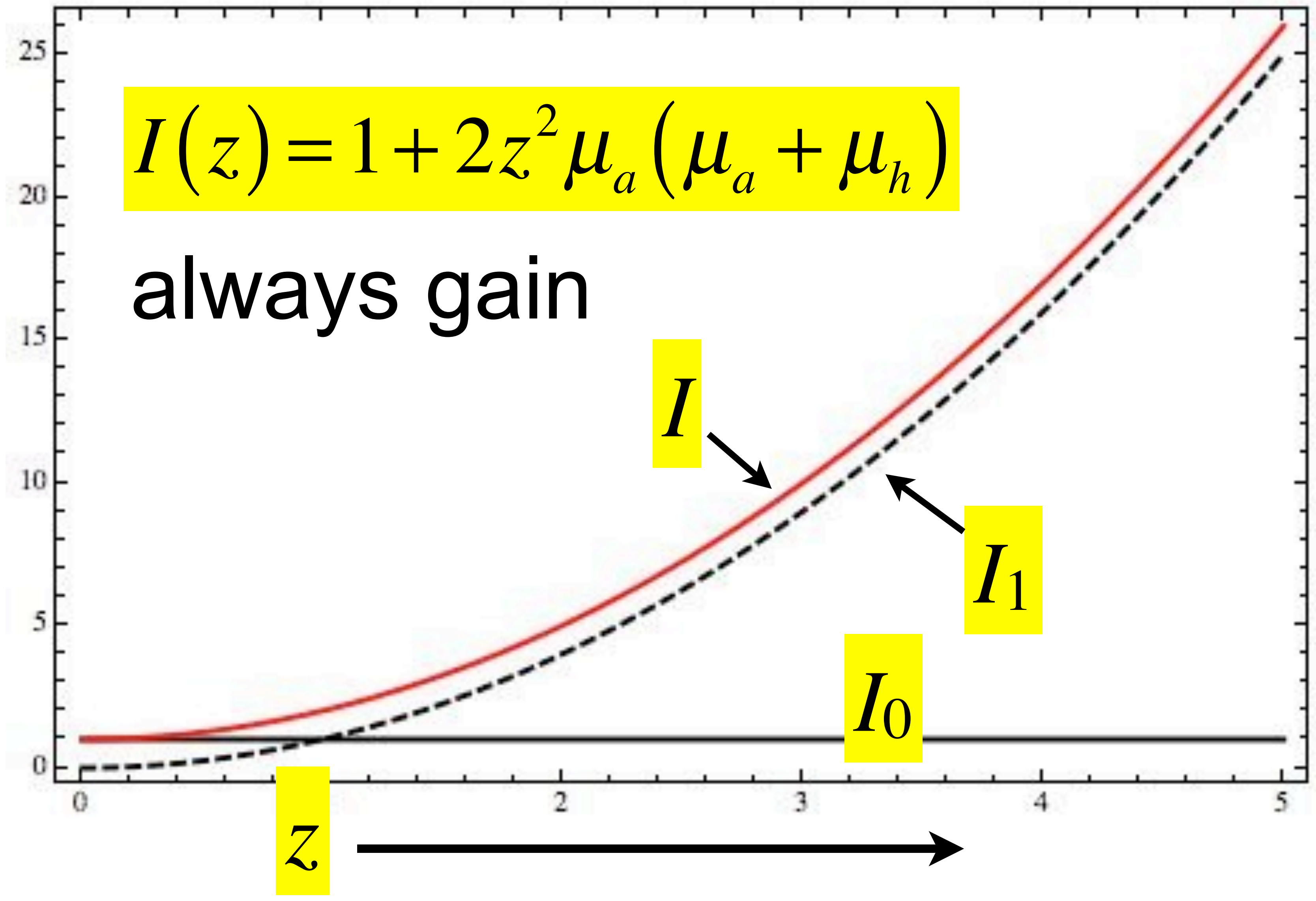

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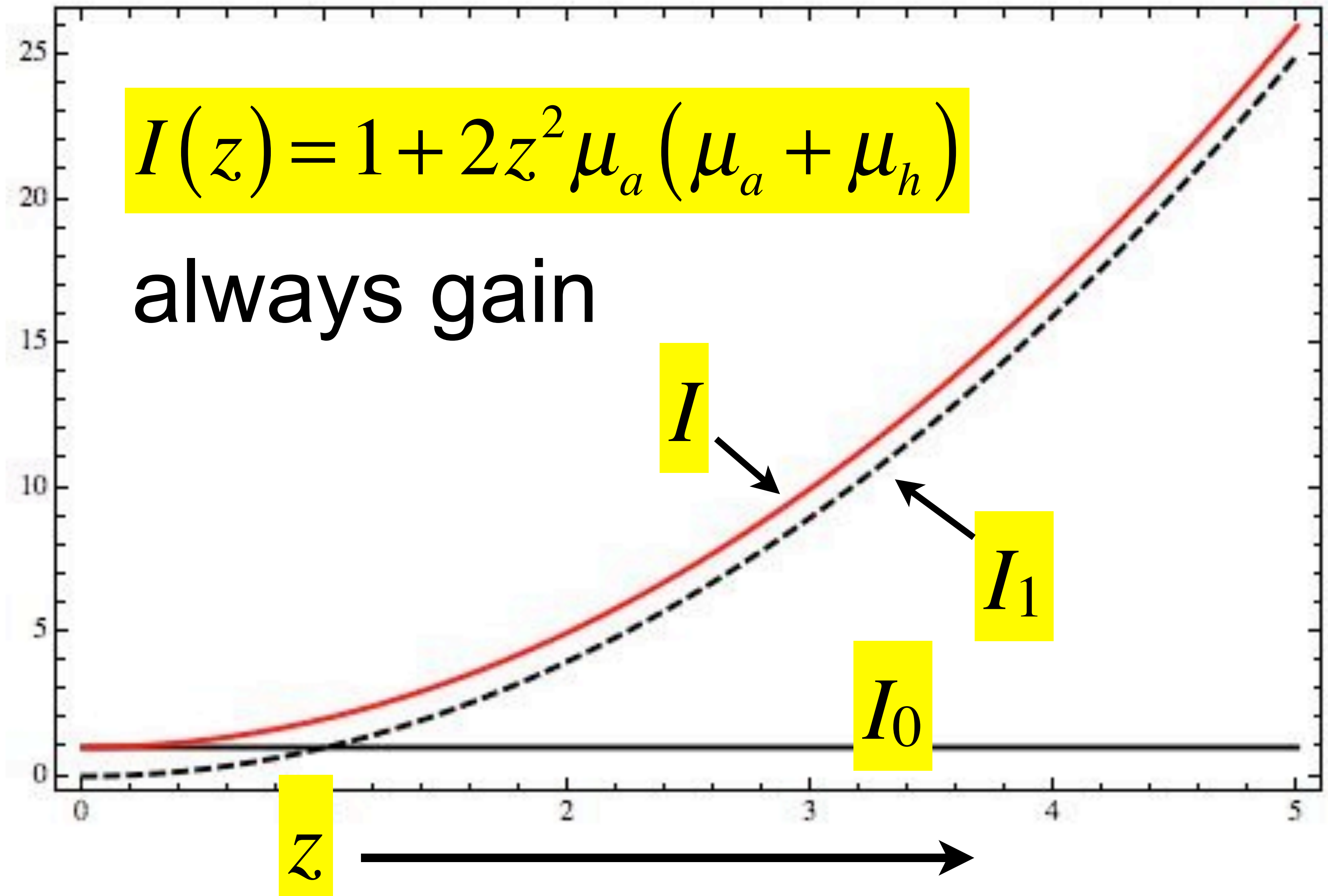

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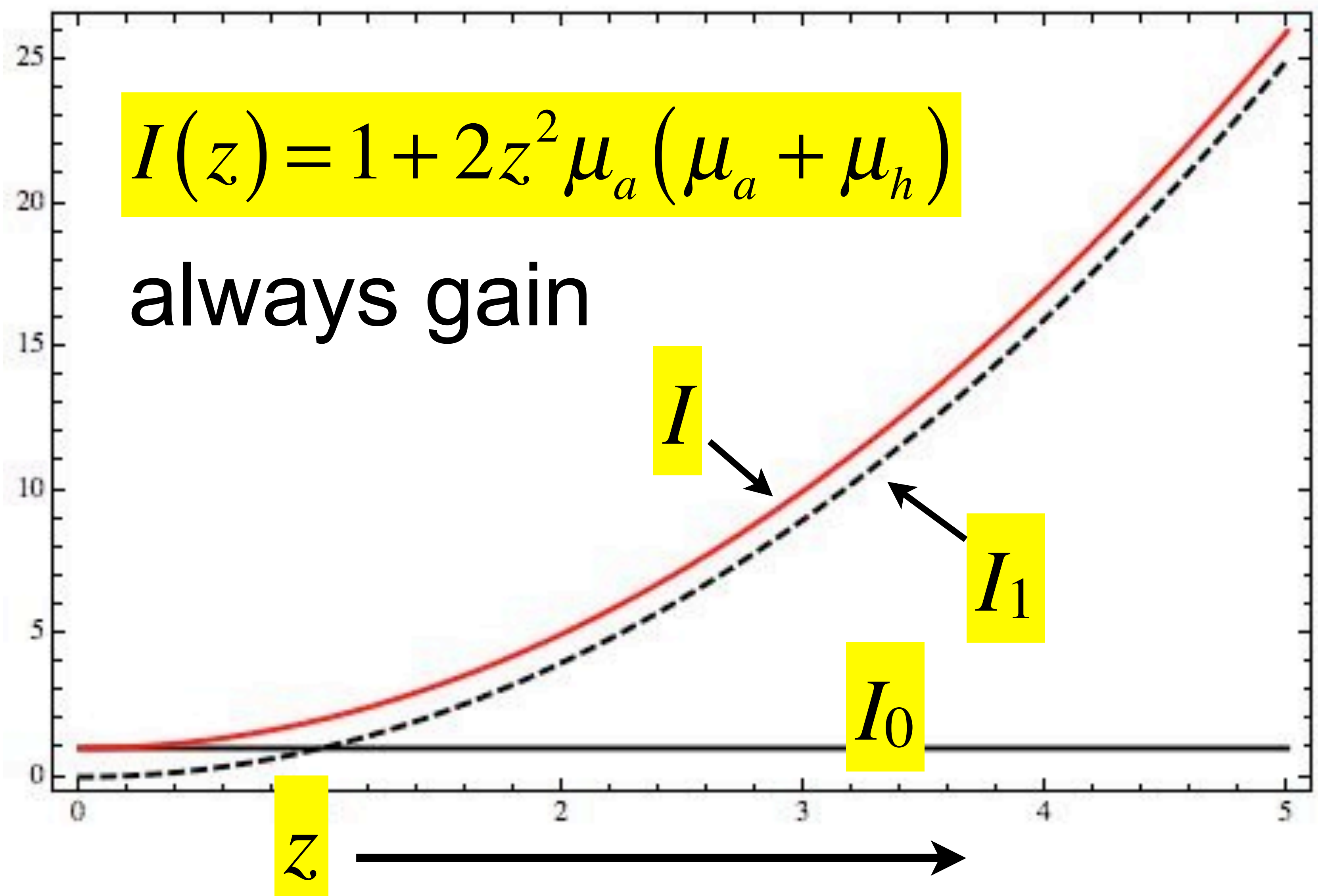

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ghost of departed eigenvector

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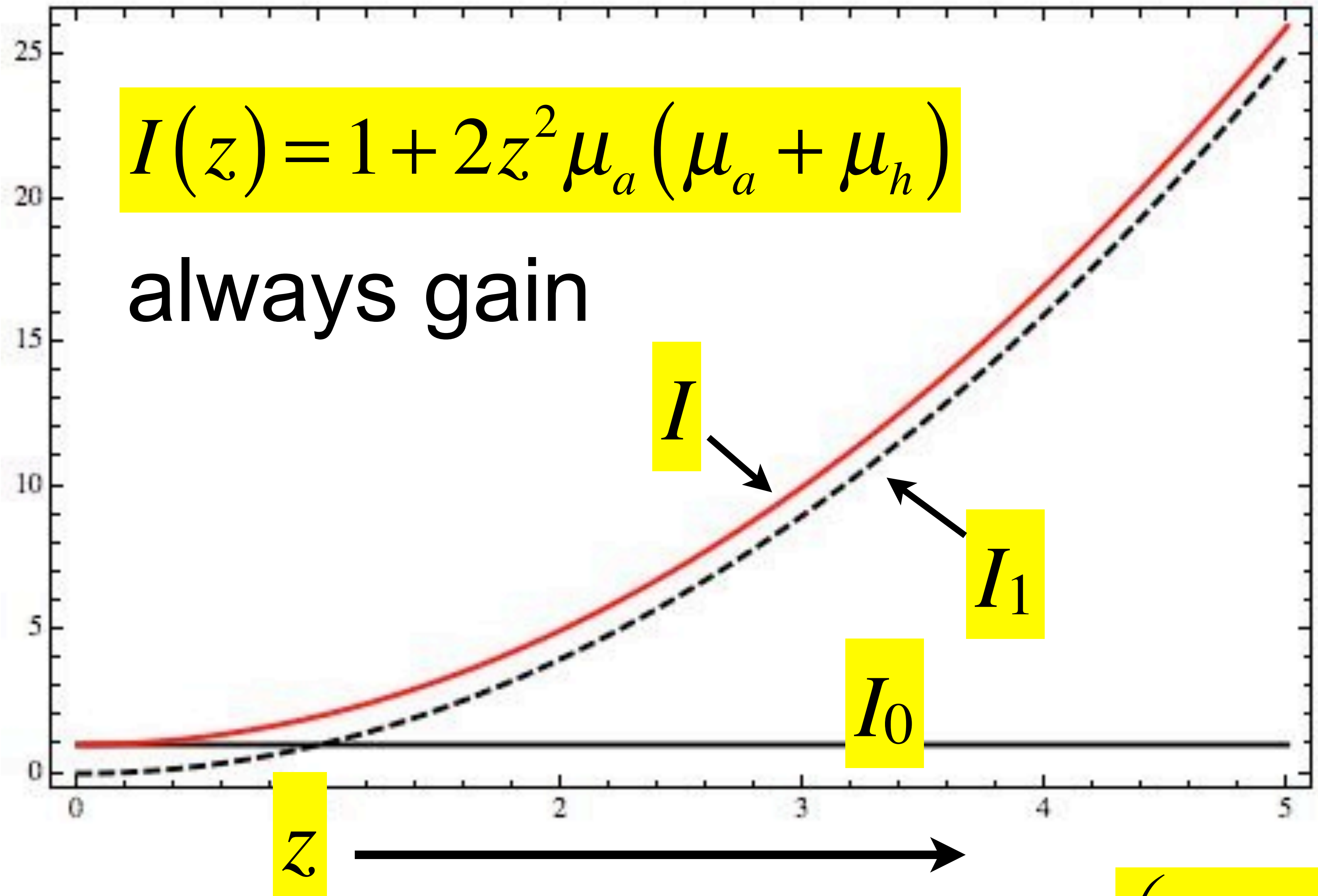
$$I(z) = 1 + 2z^2\mu_a(\mu_a + \mu_h)$$

always gain



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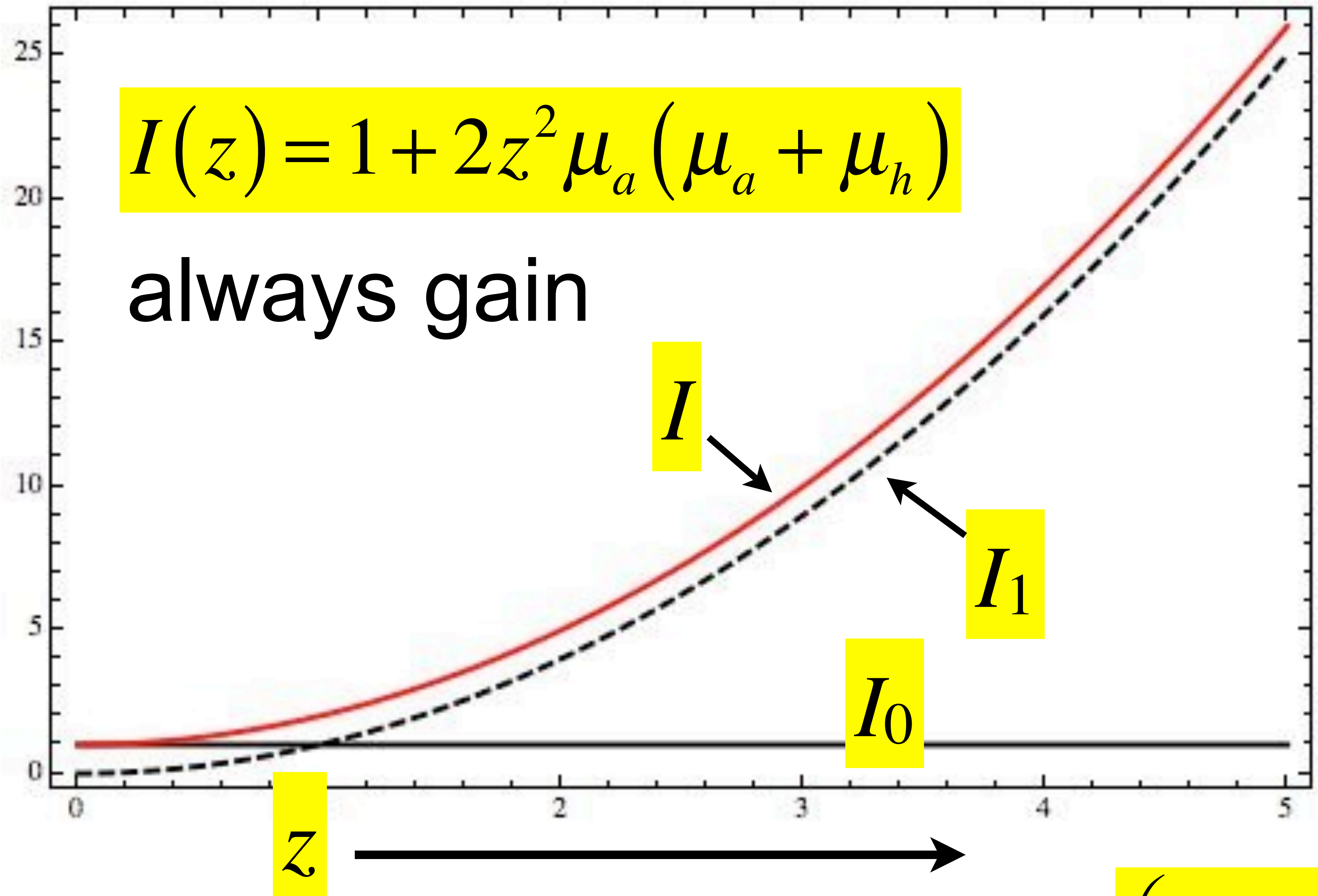

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universal NH phenomenon, not restricted to PT

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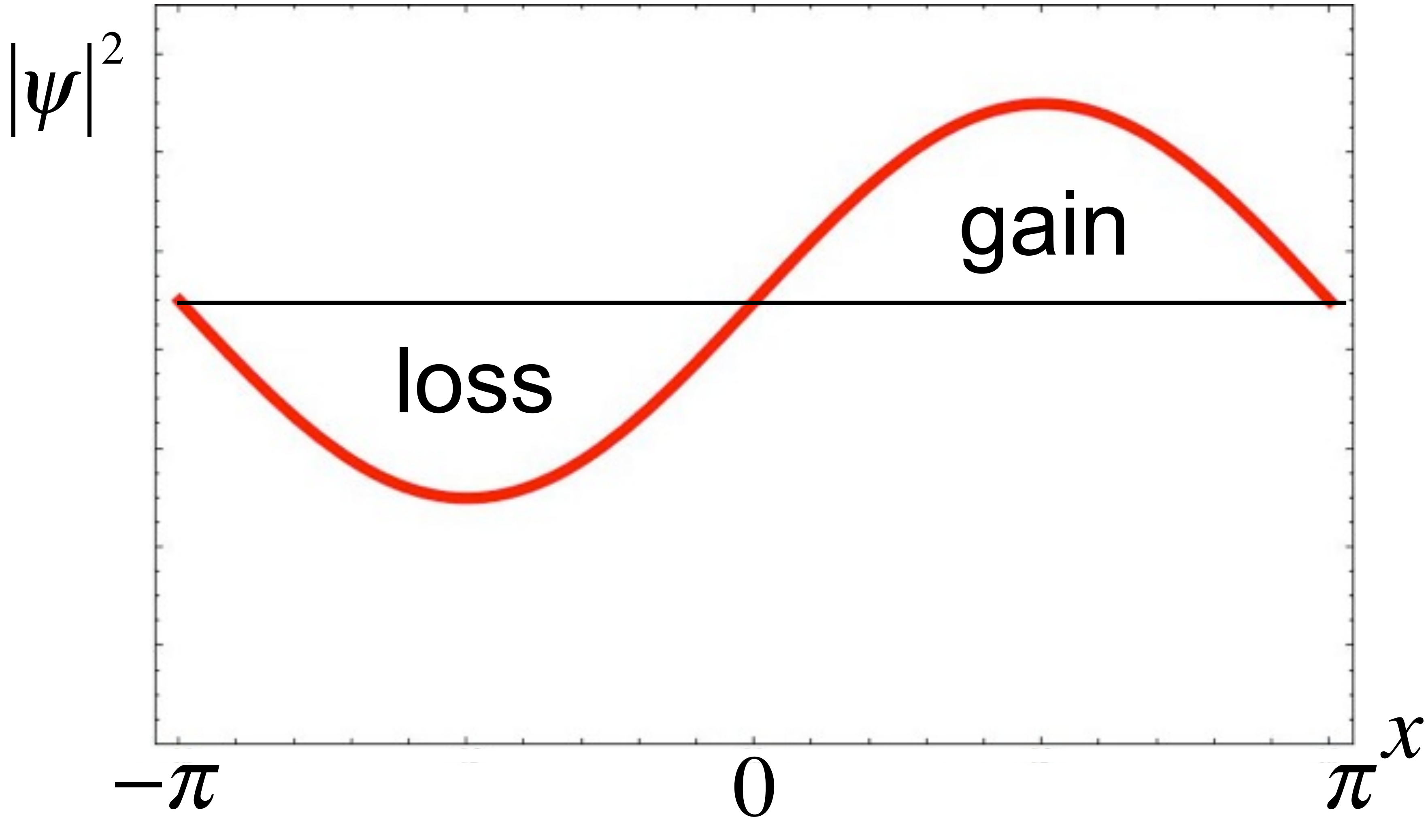

wave concentrated in gain region

Pancharatnam 1955

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in optics NHD= 'singular axis' in direction space

in optics, 2x2 dielectric matrix depending on direction, eigenvectors=polarization states

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Pancharatnam 1955: wrong! - the polarization will slowly rotate into the one that does propagate

explicitly

$$\begin{pmatrix} a_0(z) \\ a_1(z) \end{pmatrix} = \exp(-Az)\exp\left(-\mathrm{i}z\sqrt{\delta^2+\tfrac{1}{4}}\right)\times$$

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orthogonal incident polarization

polarization that propagates

explicitly

overall decay because crystal is absorbing: NH not PT, but ***the same degeneracy phenomenon***

$$\begin{pmatrix} a_0(z) \\ a_1(z) \end{pmatrix} = \boxed{\exp(-Az)} \exp\left(-\mathrm{i}z\sqrt{\delta^2+\tfrac{1}{4}}\right) \times \left[\boxed{\begin{pmatrix} \sqrt{\mu_a+\mu_h} \\ \sqrt{\mu_a-\mu_h} \end{pmatrix}} - 2\mathrm{i}z\mu_a \boxed{\begin{pmatrix} -\sqrt{\mu_a-\mu_h} \\ \sqrt{\mu_a+\mu_h} \end{pmatrix}} \right]$$

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$$\mathrm{i}\cos^2 x = \tfrac{1}{2}\mathrm{i} + \tfrac{1}{2}\mathrm{i}\sin 2\xi \quad \left(\xi = x + \tfrac{1}{4}\pi\right)$$

Pancharatnam's lossy crystal is an example of 'NH essentially PT', i.e. eigenvalues on a line parallel to the real axis: shifted to complex by absorption

another example: Zeilinger et al's (1996) atoms diffracted by light

with zero detuning, optical potential seen by atoms is proportional to $\mathrm{i}\cos^2 x$

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nonuniform loss — uniform loss — PT, i.e. gain balancing loss

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